MA 114 Worksheet #29: Review for Exam 04

This review worksheet covers only material discussed since Exam III.

To review for your final exam, be sure to study the material from Exams I, II, and III and the review sheets for these exams.

1. Identify and graph the conic section given by each of the following equations. Where applicable, find the foci.
   (a) $x^2 = 4y - 2y^2$
   (b) $x^2 + 3y^2 + 2x - 12y + 10 = 0$

2. Solve the initial value problem

$$\frac{dy}{dx} = \frac{x \sin x}{y}, \quad y(0) = -1$$

   It will help to know that

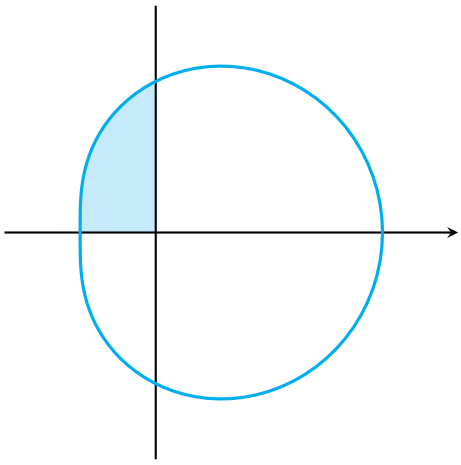
$$\int x \sin x \, dx = \sin x - x \cos x + C$$

3. By converting to Cartesian coordinates, identify and graph the curve $r^2 \sin 2\theta = 1$ (It may help to remember the identity $\sin 2\theta = 2 \sin \theta \cos \theta$).

4. Draw a direction field for the differential equation $y' = y(1-y)$. What are the equilibria? Classify each as stable or unstable.

5. Find the slope of the tangent line to the curve $r = 2 \cos \theta$ at $\theta = \pi/3$.

6. Find the area of the region shown.



$r = 2 + \cos \theta$

7. Find the exact length of the polar curve $r = \theta^2$ for $0 \leq \theta \leq 2\pi$.

8. Use Euler's method with step size 0.1 to estimate $y(0.5)$, where $y(x)$ is the solution of the initial-value problem $y' = y + xy$, $y(0) = 1$.

9. Use Euler's method with step size 0.2 to estimate $y(1)$, where $y(x)$ is the solution of the initial-value problem $y' = x^2y - \frac{1}{2}y^2$, $y(0) = 1$.

10. Solve the following differential equations.
    (a) $\dfrac{dy}{dx} = 3x^2y^2$
    (b) $xyy' = x^2 + 1$
    (c) $\dfrac{dy}{dx} + e^{x+y} = 0$

11. (a) Solve the differential equation $y' = 2x\sqrt{1 - y^2}$.
    (b) Solve the initial-value problem $y' = 2x\sqrt{1 - y^2}$, $y(0) = 0$, and graph the solution.
    (c) Does the initial-value problem $y' = 2x\sqrt{1 - y^2}$, $y(0) = 2$, have a solution? Explain.