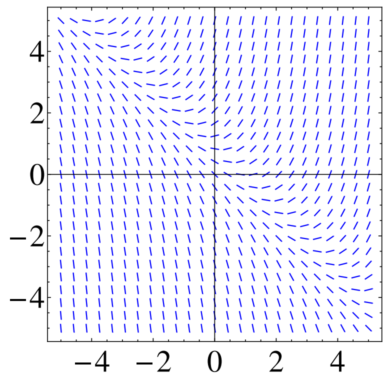
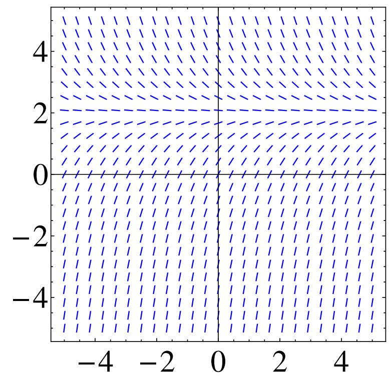
MA 114 Worksheet #28: Direction fields, Separable differential equations

1. Match the differential equation with its slope field. Give reasons for your answer.

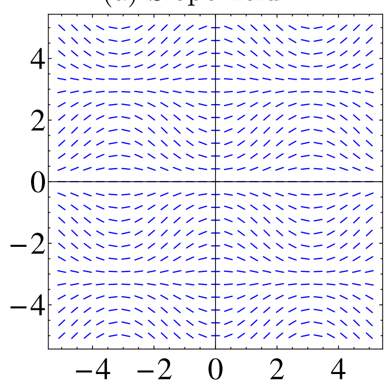
$$y' = 2 - y \qquad y' = x(2-y) \qquad y' = x + y - 1 \qquad y' = \sin(x)\sin(y)$$



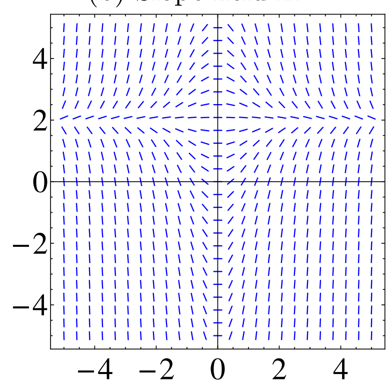
(a) Slope field I



(b) Slope field II



(c) Slope Field III



(d) Slope field IV

Figure 1: Slope fields for Problem 1

2. Use slope field labeled IV to sketch the graphs of the solutions that satisfy the given initial conditions.

$$y(0) = -1, \quad y(0) = 0, \quad y(0) = 1.$$

3. Sketch the slope field of the differential equation. Then use it to sketch a solution curve that passes through the given point.

   (a) $y' = y^2$, $(1, 1)$

   (b) $y' = y - 2x$, $(1, 0)$

   (c) $y' = xy - x^2$, $(0, 1)$

4. Consider the autonomous (depends only on y and its derivatives) differential equation $y' = y^2(3-y)(y+1)$. Without solving the differential equation, determine the value of $\lim_{t\to\infty} y(t)$, where the initial value is
   (a) $y(0) = 1$,
   (b) $y(0) = 4$,
   (c) $y(0) = -4$.

5. Use Euler's method with step size 0.5 to compute the approximate y-values, y_1, y_2, y_3, and y_4 of the solution of the initial-value problem $y' = y - 2x$, $y(1) = 0$.

6. Use separation of variables to find the general solutions to the following differential equations.
   (a) $y' + 4xy^2 = 0$
   (b) $\sqrt{1-x^2}\,y' = xy$
   (c) $(1+x^2)y' = x^3 y$
   (d) $y' = 3y - y^2$