## MA 114 Worksheet \#27: Differential equations

1. (a) Is $y=\sin (3 x)+2 e^{4 x}$ a solution to the differential equation $y^{\prime \prime}+9 y=50 e^{4 x}$ ? Explain why or why not.
(b) Explain why every solution of $d y / d x=y^{2}+6$ must be an increasing function.
(c) What does is mean to say that a differential equation is linear or nonlinear?
2. Find all values of $\alpha$ so that $y(x)=e^{\alpha x}$ is a solution of the differential equation $y^{\prime \prime}+y^{\prime}-$ $12 y=0$.
3. A tank has pure water flowing into it at 10 liters $/ \mathrm{min}$. The contents of the tank are kept thoroughly mixed, and the contents flow out at 10 liters $/ \mathrm{min}$. Salt is added to the tank at the rate of $0.1 \mathrm{~kg} / \mathrm{min}$. Initially, the tank contains 10 kg of salt in 100 liters of water. Formulate an initial value problem (that is, a differential equation along with initial conditions) whose solution is the quantity of salt in the tank at any time $t$. Do not solve the initial value problem.
4. Consider a tank with 200 liters of salt-water solution. A salt-water solution, with a concentration of 2 grams per liter, pours into the tank at a rate of 4 liters per minute. The well-mixed solution in the tank pours out at the same rate of 4 liters/minute. Write a differential equation expressing the rate of change in the concentration, $c(t)$, of salt in the tank. Do not solve.
