MA 114 Worksheet #25: Calculus with polar coordinates

1. Find dy/dx for the following polar curves.

   (a) $r = 2\cos\theta + 1$ (b) $r = 1/\theta$ (c) $r = 2e^{-\theta}$

2. In each of the following, compute the slope of the tangent line at the given point. Then sketch the curve and the tangent line.

   (a) $r = \sin\theta$ at $\theta = \pi/3$. (b) $r = 1/\theta$ at $\theta = \pi/2$.

3. (a) Give the formula for the area of region bounded by the polar curve $r = f(\theta)$ from $\theta = a$ to $\theta = b$. Give a geometric explanation of this formula.

   (b) Give the formula for the length of the polar curve $r = f(\theta)$ from $\theta = a$ to $\theta = b$.

   (c) Use these formulas to establish the formulas for the area and circumference of a circle.

4. Find the slope of the tangent line to the polar curve $r = \theta^2$ at $\theta = \pi$.

5. Find the point(s) where the tangent line to the polar curve $r = 2 + \sin\theta$ is horizontal.

6. Find the area enclosed by one leaf of the curve $r = \sin 2\theta$.

7. Find the arc length of one leaf of the curve $r = \sin 2\theta$.

8. Find the area of the region bounded by $r = \cos\theta$ for $\theta = 0$ to $\theta = \pi/4$.

9. Find the area of the region that lies inside both the curves $r = \sqrt{3}\sin\theta$ and $r = \cos\theta$.

10. Find the area in the first quadrant that lies inside the curve $r = 2\cos\theta$ and outside the curve $r = 1$.

11. Find the length of the curve $r = \theta^2$ for $0 \le \theta \le 2\pi$.

12. Write down an integral expression for the length of the curve $r = \sin\theta + \theta$ for $0 \le \theta \le \pi$ but do not compute the integral.

13. Consider the sequence of circles, C_n, defined by the equations $x^2 + \left(y + \frac{1}{\sqrt{n}}\right)^2 = \frac{1}{n}$. Define a_n as the area of circle C_n and b_n as the area between circles C_n and C_{n+1}.

    (a) Sketch the picture of this infinite sequence of circles.

    (b) Does $\sum_{n=1}^{\infty} a_n$ converge?

    (c) Does $\sum_{n=1}^{\infty} b_n$ converge?

    (d) Define the circles D_n by the equations $x^2 + \left(y + \frac{1}{n}\right)^2 = \frac{1}{n^2}$ with d_n as the area of D_n. Does $\sum_{n=1}^{\infty} d_n$ converge?