MA 114 Worksheet #18: Volumes I

1. If a solid has a cross-sectional area given by the function $A(x)$, what integral should be evaluated to find the volume of the solid?

2. Calculate the volume of the following solid. The base is a square, one of whose sides is the interval $[0, l]$ along the x-axis. The cross sections perpendicular to the x-axis are rectangles of height $f(x) = x^2$.

3. Calculate the volume of the following solid. The base is the region enclosed by $y = x^2$ and $y = 3$. The cross sections perpendicular to the y-axis are squares.

4. The base of a certain solid is the triangle with vertices at $(-10, 5)$, $(5, 5)$, and the origin. Cross-sections perpendicular to the y-axis are squares. Find the volume of the solid.

5. Calculate the volume of the following solid. The base is a circle of radius r centered at the origin. The cross sections perpendicular to the x-axis are squares.

6. Calculate the volume of the following solid. The base is the parabolic region $\{(x, y) \mid x^2 \leq y \leq 4\}$. The cross sections perpendicular to the y-axis are right isosceles triangles whose hypotenuse lies in the region.

7. Sketch a solid whose volume is given by the integral

$$\pi \int_0^1 (y^2 + 1)^2 - 1 \, dy.$$

8. For each of the following, use disks or washers to find the an integral expression for the volume of the region. Evaluate the integrals for parts (a) and (d).
   (a) R is region bounded by $y = 1 - x^2$ and $y = 0$; about the x-axis.
   (b) R is region bounded by $y = \dfrac{1}{x}$, $x = 1$, $x = 2$, and $y = 0$; about the x-axis.
   (c) R is region bounded by $x = 2\sqrt{y}$, $x = 0$, and $y = 9$; about the y-axis.
   (d) R is region bounded by $y = 1 - x^2$ and $y = 0$; about the line $y = -1$.
   (e) Between the regions in part (a) and part (d), which volume is bigger? Why?
   (f) R is region bounded by $y = e^{-x}$, $y = 1$, and $x = 2$; about the line $y = 2$.
   (g) R is region bounded by $y = x$ and $y = \sqrt{x}$; about the line $x = 2$.

9. Find the volume of the cone obtained by rotating the region under the segment joining $(0, h)$ and $(r, 0)$ about the y-axis.

10. The torus is the solid obtained by rotating the circle $(x-a)^2 + y^2 = b^2$ around the y-axis (assume that $a > b$). Show that it has volume $2\pi^2 ab^2$.
    [Hint: Draw a picture, set up the problem and evaluate the integral by interpreting it as the area of a circle.]