

## MA 114 Worksheet #16: Review for Exam 02

1. List the first five terms of the sequence:

$$(a) a_n = \frac{(-1)^n n}{n! + 1}$$

$$(b) a_1 = 6, a_{n+1} = \frac{a_n}{n}.$$

2. Determine whether the sequence converges or diverges. If it converges, find the limit.

$$(a) a_n = 3^n 7^{-n}$$

$$(c) a_n = \frac{\ln n}{\ln 2n}$$

$$(b) a_n = \frac{(-1)^{n+1} n}{n + \sqrt{n}}$$

$$(d) a_n = \frac{\cos^2 n}{2^n}$$

3. Explain what it means to say that  $\sum_{n=1}^{\infty} a_n = 2$ .

4. Determine whether the geometric series is convergent or divergent. If it is convergent, find its sum.

$$(a) \sum_{n=1}^{\infty} \frac{(-4)^{n-1}}{3^n}$$

$$(b) \sum_{n=1}^{\infty} \frac{6 \cdot 2^{n-1}}{3^n}$$

5. Determine whether the given series converges or diverges and state which test you used.

$$(a) \sum_{n=2}^{\infty} \frac{1}{n \ln n}$$

$$(e) \sum_{n=1}^{\infty} \frac{9^n}{9n}$$

$$(b) \sum_{n=1}^{\infty} \frac{7\sqrt{n}}{5n^{3/2} + 3n - 2}$$

$$(f) \sum_{n=1}^{\infty} (-1)^{n+1} n e^{-n}$$

$$(c) \sum_{n=1}^{\infty} n! e^{-8n}$$

$$(g) \sum_{n=1}^{\infty} (-1)^{n-1} \arctan n$$

$$(d) \sum_{n=1}^{\infty} \left( \frac{\ln n}{5n + 7} \right)^n$$

6. Determine whether the given series is absolutely convergent or conditionally convergent or divergent.

$$(a) \sum_{n=1}^{\infty} \frac{(-1)^n}{5n + 1}$$

$$(c) \sum_{n=1}^{\infty} (-1)^n \cos \left( \frac{1}{n^2} \right)$$

$$(b) \sum_{n=1}^{\infty} \frac{(-1)^n}{n^3 + 1}$$

$$(d) \sum_{n=1}^{\infty} \frac{(n!)^n}{n^{4n}}$$

7. Find the radius and interval of convergence of the series.

(a) 
$$\sum_{n=1}^{\infty} \frac{x^n}{n^4 4^n}$$

(b) 
$$\sum_{n=1}^{\infty} \frac{(-1)^n x^n}{n^2}$$

(c) 
$$\sum_{n=1}^{\infty} \frac{(5x - 4)^n}{n^3}$$

8. Find a power series representation for the function and determine its radius of convergence.

(a) 
$$f(x) = \frac{5}{1 - 4x^2}$$

(c) 
$$f(x) = \frac{3}{2 + 2x}$$

(b) 
$$f(x) = \frac{x^2}{x^4 + 16}$$

(d) 
$$f(x) = e^{-x^2}$$

9. Using the formula

$$\ln(1 + x) = \int_0^x \frac{1}{1 + t} dt$$

find a power series for  $\ln(1 + x)$  and state its radius of convergence.

10. Use the Maclaurin series for  $\cos(x)$  to compute

$$\lim_{x \rightarrow 0} \frac{1 - \cos(x)}{x^2}.$$