MA 114 Worksheet #12: Alternating Series, Absolute Convergence, & Conditional Convergence

1. (a) Let $a_n = \dfrac{n}{3n+1}$. Does $\{a_n\}$ converge? Does $\sum_{n=1}^{\infty} a_n$ converge?
   (b) Give an example of a divergent series $\sum_{n=1}^{\infty} a_n$ where $\lim_{n\to\infty} a_n = 0$.
   (c) Does there exist a convergent series $\sum_{n=1}^{\infty} a_n$ which satisfies $\lim_{n\to\infty} a_n \neq 0$? Explain.
   (d) When does a series converge absolutely? When does a series converge conditionally?
   (e) State the alternating series test.
   (f) Prove that the alternating harmonic series $\displaystyle\sum_{n=1}^{\infty} \frac{(-1)^n}{n}$ converges.
   (g) State the Alternating Series Estimation Theorem.

2. Test the following series for convergence or divergence.
   (a) $\displaystyle\sum_{n=1}^{\infty} (-1)^n \frac{\sqrt{n}}{1+2n}$
   (b) $\displaystyle\sum_{n=2}^{\infty} (-1)^n \frac{1}{\ln n}$
   (c) $\displaystyle\sum_{n=1}^{\infty} \frac{\cos n\pi}{n^{2/3}}$
   (d) $\displaystyle\sum_{n=1}^{\infty} \frac{3^n}{4^n+5^n}$
   (e) $\displaystyle\sum_{n=2}^{\infty} (-1)^n \frac{n}{\ln n}$
   (f) $\displaystyle\sum_{n=1}^{\infty} \left(\frac{-5}{18}\right)$

3. Use the Alternating Series Estimation Theorem to estimate the sum correct to four decimal places.
   (a) $\displaystyle\sum_{n=1}^{\infty} \frac{(-0.8)^n}{n!}$
   (b) $\displaystyle\sum_{n=1}^{\infty} (-1)^{n-1} \frac{n}{8^n}$

4. Approximate the sum of the series $\displaystyle\sum_{n=1}^{\infty} (-1)^n \frac{1}{(2n)!}$ correct to four decimal places; *i. e.* so that $|\text{error}| < 0.00005$.