MA 114 Worksheet #10: Series and The Integral Test

1. Identify the following statements as true or false and explain your answers.
   (a) If the sequence of partial sums of an infinite series is bounded the series converges.
   (b) $\sum_{n=1}^{\infty} a_n = \lim_{n\to\infty} a_n$ if the series converges.
   (c) $\sum_{n=1}^{\infty} a_n = \sum_{n=0}^{\infty} a_{n+1}$ if both series converge.
   (d) If c is a nonzero constant and if $\sum_{n=1}^{\infty} ca_n$ converges then so does $\sum_{n=1}^{\infty} a_n$.
   (e) A finite number of terms of an infinite series may be changed without affecting whether or not the series converges.
   (f) Every infinite series with only finitely many nonzero terms converges.

2. Write the following in summation notation:
   (a) $\frac{1}{9} + \frac{1}{16} + \frac{1}{25} + \frac{1}{36} + \ldots$
   (b) $1 - \frac{1}{3} + \frac{1}{5} - \frac{1}{7} + \ldots$

3. Calculate S_3, S_4, and S_5 and then find the sum of the telescoping series $S = \sum_{n=1}^{\infty} \left(\frac{1}{n+1} - \frac{1}{n+2} \right)$.

4. Use the formula for the sum of a geometric series to find the sum or state that the series diverges and why:
   (a) $\frac{1}{1} + \frac{1}{8} + \frac{1}{8^2} + \ldots$
   (b) $\sum_{n=0}^{\infty} \left(\frac{\pi}{e} \right)^n$

5. Use the Integral Test to determine if the following series converge or diverge:
   (a) $\sum_{n=0}^{\infty} \frac{1}{1+n^2}$
   (b) $\sum_{n=1}^{\infty} n^2 e^{-n^3}$
   (c) $\sum_{n=2}^{\infty} \frac{n}{(n^2+2)^{3/2}}$

6. Show that the infinite series $\sum_{n=1}^{\infty} \frac{1}{n^p}$ converges if $p > 1$ and diverges otherwise by Integral Test.