

MA 114 Worksheet #09: Recursive Sequences and Series

1. Write out the first five terms of

(a) $a_0 = 0$, $a_1 = 1$ and $a_{n+1} = 3a_{n-1} + a_n^2$.

(b) $a_1 = 6$, $a_{n+1} = \frac{a_n}{n}$.

(c) $a_1 = 2$, $a_2 = 1$, and $a_{n+1} = a_n - a_{n-1}$.

2. (a) For what values of x does the sequence $\{x^n\}_{n=1}^{\infty}$ converge?

(b) For what values of x does the sequence $\{n^x\}_{n=1}^{\infty}$ converge?

(c) If $\lim_{n \rightarrow \infty} b_n = \sqrt{2}$, find $\lim_{n \rightarrow \infty} b_{n-3}$.

3. Determine if each of the following sequences is convergent. If the sequence is convergent, find the limit.

(a) $a_n = 1 + (-1)^n + 2^{-n}$

(c) $c_n = \frac{1}{2 + \sin(1/n)}$

(b) $b_n = \frac{2^n}{2^n - 1}$

4. Newton's method for solving the function $f(x) = x^2 - 2 = 0$ gives a recursive sequence $a_{n+1} = \frac{1}{2}(a_n + 2/a_n)$. Suppose that $\lim_{n \rightarrow \infty} a_n = A$ exists. Find an equation satisfied by A . What are the possible values of A .

5. (a) Determine whether the sequence defined as follows is convergent or divergent:

$$a_1 = 1 \quad a_{n+1} = 4 - a_n \quad \text{for } n > 1.$$

(b) What happens if the first term is $a_1 = 2$?

6. A fish farmer has 5000 catfish in his pond. The number of catfish increases by 8% per month and the farmer harvests 300 catfish per month.

(a) Show that the catfish population P_n after n months is given recursively by

$$P_n = 1.08P_{n-1} - 300 \quad P_0 = 5000.$$

(b) How many catfish are in the pond after six months?