MA 114 Worksheet #07: Sequences

1. (a) Give the precise definition of a **sequence**.

   (b) What does it mean to say that $\lim_{x\to a} f(x) = L$ when $a = \infty$? Does this differ from $\lim_{n\to\infty} f(n) = L$? Why or why not?

   (c) What does it means for a sequence to converge? Explain your idea, not just the definition in the book.

   (d) Sequences can diverge in different ways. Describe two distinct ways that a sequence can diverge.

   (e) Give two examples of sequences which converge to 0 and two examples of sequences which converges to a given number $L \neq 0$.

2. Write the first four terms of the sequences with the following general terms:

   (a) $\dfrac{n!}{2^n}$

   (b) $\dfrac{n}{n+1}$

   (c) $(-1)^{n+1}$

   (d) $\{a_n\}_{n=1}^{\infty}$ where $a_n = \dfrac{3}{n}$.

   (e) $\{a_n\}_{n=1}^{\infty}$ where $a_n = 2^{-n} + 2$.

   (f) $\{b_k\}_{k=1}^{\infty}$ where $b_k = \dfrac{(-1)^k)}{k^2}$.

3. Find a formula for the nth term of each sequence.

   (a) $\left\{\dfrac{1}{1}, -\dfrac{1}{8}, \dfrac{1}{27}, -\dfrac{1}{64}, \dots\right\}$

   (b) $\left\{1, \dfrac{1}{2}, \dfrac{1}{4}, \dfrac{1}{8}, \dfrac{1}{16}, \dots\right\}$

   (c) $\{1, 0, 1, 0, 1, 0, \dots\}$

   (d) $\left\{-\frac{1}{2}, \frac{2}{3}, -\frac{3}{4}, \frac{4}{5}, -\frac{5}{6}, \dots,\right\}$

4. Suppose that a sequence $\{a_n\}$ is bounded above and below. Does it converge? If not, find a counterexample.

5. The limit laws for sequences are the same as the limit laws for functions. Suppose you have sequences $\{a_n\}$, $\{b_n\}$ and $\{c_n\}$ with $\lim_{n\to\infty} a_n = 15$, $\lim_{n\to\infty} b_n = 0$ and $\lim_{n\to\infty} c_n = 1$. Use the limit laws of sequences to answer the following questions.

   (a) Does the sequence $\left\{\dfrac{a_n \cdot c_n}{b_n + 1}\right\}_{n=1}^{\infty}$ converge? If so, what is its limit?

   (b) Does the sequence $\left\{\dfrac{a_n + 3 \cdot c_n}{2 \cdot b_n + 2}\right\}_{n=1}^{\infty}$ converge? If so, what is its limit?