MA 114 Worksheet #03: Trigonometric Substitution

1. Use the trigonometric substitution $x = \sin(u)$ to find $\displaystyle\int \frac{1}{\sqrt{1-x^2}}\,dx$.

   Remark: This exercise verifies one of the basic anti-derivatives we learned in Calculus I. On an exam, you would be expected to know this anti-derivative and would not be expected to show work to evaluate the anti-derivative by substitution.

2. Compute the following integrals:

   (a) $\displaystyle\int_0^2 \frac{u^3}{\sqrt{16-u^2}}du$

   (b) $\displaystyle\int \frac{1}{x^2\sqrt{25-x^2}}dx$

   (c) $\displaystyle\int \frac{x^2}{\sqrt{9-x^2}}\,dx$

   (d) $\displaystyle\int \frac{x^3}{\sqrt{4+x^2}}\,dx$

   (e) $\displaystyle\int \frac{1}{(1+x)^2}\,dx$

   (f) $\displaystyle\int_0^3 \frac{x}{\sqrt{36-x^2}}dx.$

3. Evaluate the following integrals. One may be easily evaluated by substitution $u = 1+x^2$ and for the other use an appropriate trigonometric subsitution.

$$\int \frac{\sqrt{1+x^2}}{x}\,dx \qquad \int \frac{x}{\sqrt{1+x^2}}\,dx$$

4. (a) Evaluate the integral $\displaystyle\int_0^r \sqrt{r^2-x^2}\,dx$ using trigonometric substitution.

   (b) Use your answer to part a) to verify the formaula for the area of a circle of radius r.

5. Let $r > 0$. Consider the identity

$$\int_0^s \sqrt{r^2-x^2}\,dx = \frac{1}{2}r^2 \arcsin{(s/r)} + \frac{1}{2}s\sqrt{r^2-s^2}$$

   where $0 \le s \le r$.

   (a) Plot the curves $y = \sqrt{r^2-x^2}$, $x = s$, and $y = \dfrac{x}{s}\sqrt{r^2-x^2}$.

   (b) Using part (a), verify the identity geometrically.

   (c) Verify the identity using trigonometric substitution.