Exercise 1: Write out the general form for the partial fraction decomposition.

c) $\dfrac{x}{(x^2+1)(x+1)(x+2)} = \dfrac{Ax+B}{x^2+1} + \dfrac{C}{x+1} + \dfrac{D}{x+2}$

$$\frac{(Ax+B)(x+1)(x+2) + C(x^2+1)(x+2) + D(x^2+1)(x+1)}{(x^2+1)(x+1)(x+2)} = \frac{x}{(x^2+1)(x+1)(x+2)}$$

$$\frac{\left((Ax+B)(x+1)(x+2) + C\left((x^2+1)(x+2)\right) + D(x^2+1)(x+1)\right)}{(x^2+1)(x+1)(x+2)}$$

$$\frac{(Ax^2+Ax+Bx+B)(x+2) + C\overbrace{(x^3+2x^2+x+2)}^{1} + D\overbrace{(x^3+x^2+x+1)}^{2}}{(x^2+1)(x+1)(x+2)} =$$

$$= \frac{Ax^3+Ax^2+Bx^2+Bx+2Ax^2+2Ax+2Bx+2B + C(\ldots 1 \ldots) + D(\ldots 2 \ldots)}{(x^2+1)(x+1)(x+2)}$$

$$\frac{Ax^3 + 2Ax^2 + Ax^2 + 2AX + Bx^2 + 2BX + BX + 2B + Cx^3 + 2Cx^2 + Cx + 2C + Dx^3 + Dx + Dx + D}{(x^2+1)(x+1)(x+2)}$$

$$= \frac{1 \cdot x + 0 \cdot x^2 + 0 \cdot x^3 + 0}{(x^2+1)(x+1)(x+2)}$$

$$= \frac{x^3 \overbrace{(A + C + D)}^{0} + x^2 \overbrace{(3A + B + 2C + D)}^{0} + x \overbrace{(2A + 3B + C + D)}^{1} + \overbrace{(2B + 2C + D)}^{0}}{= \parallel = \parallel =}$$

$$\begin{cases} A+C+D=0 \\ 3A+B+2C+D=0 \\ 2A+3B+C+D=1 \\ 2B+2C+D=0 \end{cases}$$

$\Leftarrow$ $\boxed{A=-C-D}$

$\Leftarrow$ $\boxed{B=\dfrac{-2C-D}{2}}$

$$\Rightarrow \begin{cases} 3(-C-D)+\dfrac{-2C-D}{2}+2C+D=0 \\ 2(-C-D)+3\,\dfrac{-2C-D}{2}+C+D=1 \end{cases}$$

$$\begin{cases} -3C-3D-C-\dfrac{D}{2}+2C+D=0 \\ -2C-2D-3C-\dfrac{3}{2}D+C+D=1 \end{cases}$$

$$\Leftrightarrow \begin{cases} -2C-2D-\dfrac{D}{2}=0 \quad \cdot(-2) \\ -4C-D-\dfrac{3}{2}D=1 \end{cases}$$

$-2\cdot\dfrac{-1}{2}=1$

$$+\begin{cases} 4C + 4D + D = 0 \\ -4C - D - \frac{3}{2}D = 1 \end{cases}$$

$$\left(-\cancel{4C} - D - \frac{3}{2}D\right) + \cancel{4C} + 5D = 0 + 1$$

$$4D - \frac{3}{2}D = 1$$

$$\frac{8-3}{2} \cdot D = 1 \Rightarrow$$

$$\frac{5}{2} \cdot D = 1$$

$$\boxed{D = \frac{2}{5}}$$

$$\boxed{-2C - 2 \cdot \frac{2}{5} - \frac{\frac{2}{5}}{2} = 0}$$

$$-2C - \frac{4}{5} - \frac{\cancel{2}}{5} \cdot \frac{1}{\cancel{2}} = 0$$

$$-2C - \frac{4}{5} - \frac{1}{5} = 0 \Rightarrow -2C - 1 = 0 \Rightarrow 2C = -1 \Rightarrow \boxed{C = -\frac{1}{2}}$$

$$A = -C - D = \frac{1}{2} - \frac{2}{5} = \frac{5-4}{10} = \boxed{\frac{1}{10} = A}$$

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c) $$\frac{x}{(x^2+1)(x+1)(x+2)} = \frac{Ax+B}{x^2+1} + \frac{C}{x+1} + \frac{D}{x+2}$$

$$\frac{(Ax+B)(x+1)(x+2) + C(x^2+1)(x+2) + D(x^2+1)(x+1)}{(x^2+1)(x+1)(x+2)} = \frac{x}{(x^2+1)(x+1)(x+2)}$$

$$\frac{\big((Ax+B)(x+1)(x+2) + C(x^2+1)(x+2) + D(x^2+1)(x+1)\big)}{(x^2+1)(x+1)(x+2)}$$

$$\frac{(Ax^2+Ax+Bx+B)(x+2) + C\overset{1}{(x^3+2x^2+x+2)} + D\overset{2}{(x^3+x^2+x+1)}}{(x^2+1)(x+1)(x+2)} =$$

$$= \frac{Ax^3+Ax^2+Bx^2+Bx+2Ax^2+2Ax+2Bx+2B+C(\ldots 1 \ldots)+D(\ldots 2 \ldots)}{(x^2+1)(x+1)(x+2)}$$

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$$\begin{cases} A + C + D = 0 \\ 3A + B + 2C + D = 0 \\ 2A + 3B + C + D = 1 \\ 2B + 2C + D = 0 \end{cases}$$

$$A = -C - D$$

$$B = \frac{-2C - D}{2}$$

$$\Rightarrow \begin{cases} 3(-C - D) + \dfrac{-2C - D}{2} + 2C + D = 0 \\ 2(-C - D) + 3\,\dfrac{-2C - D}{2} + C + D = 1 \end{cases}$$

$$\begin{cases} -3C - 3D - C - \dfrac{D}{2} + 2C + D = 0 \\ -2C - 2D - 3C - \dfrac{3}{2}D + C + D = 1 \end{cases}$$

$$\Leftrightarrow \; -2 \begin{cases} -2C - 2D - \dfrac{D}{2} = 0 \\ -4C - D - \dfrac{3}{2}D = 1 \end{cases}$$

$$-2 \cdot \frac{-1}{2} = 1$$

$$\begin{cases} 4C + 4D + D = 0 \\ -4C - D - \frac{3}{2}D = 1 \end{cases}$$

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Exercise 1:

a) $\frac{1}{x^2+3x+2}$

$x^2+3x+2=0$

Use Quadratic formula

$a=1,\ b=3,\ c=2$

$$x_{1,2}=\frac{-3\pm\sqrt{9-4\cdot 2}}{2}$$

$$=\frac{-3\pm 1}{2}\Rightarrow \quad x_1=\frac{-3+1}{2}=-1 \qquad x_2=\frac{-3-1}{2}=-2$$

In general:

$ax^2+bx+c=0$

$$x_{1,2}=\frac{-b\pm\sqrt{b^2-4\cdot a\cdot c}}{2\cdot a}$$

$$x^2+3x+2=(x-(-1))(x-(-2))$$

$$=(x+1)(x+2)$$

$$\frac{1}{x^2+3x+2} = \frac{1}{(x+1)(x+2)} = \frac{A}{x+1} + \frac{B}{x+2} =$$

$2+1 \qquad 2\cdot 1$

$$= \frac{A(x+2)+B(x+1)}{(x+1)(x+2)} = \frac{Ax+2A+Bx+B}{(x+1)(x+2)} = \frac{1+0\cdot x}{(x+1)(x+2)}$$

$$\begin{cases} A+B=0 \Rightarrow A=-B \\ 2A+B=1 \qquad -2B+B=1 \Rightarrow -B=1 \Rightarrow \boxed{B=-1} \end{cases}$$

$$A=-B=\boxed{1=A}$$

$$\boxed{\int \frac{1}{x^2+3x+2} = \int \frac{1}{x+1} + \int \frac{-1}{x+2}}$$

Exercise 3:

5) $\dfrac{x^3}{x^2-4}$

$$\begin{array}{r|l} x^3 & x^2-4 \\ \hline -(x^3-4x) & x \\ \hline 4x & \end{array}$$

$$\boxed{\frac{x^3}{x^2-4} = x + \frac{4x}{x^2-4}}$$

Worksheet 4 :

Exercise 1 : Write out the general form for the partial fraction decomposition.

c) $\dfrac{x}{(x^2+1)(x+1)(x+2)} =$

$x^2+1=0 \Rightarrow x^2=-1$

$x = \pm\sqrt{-1}$ (!!!)

$= \dfrac{A\cancel{x}+B}{x^2+1} + \dfrac{C}{x+1} + \dfrac{D}{x+2} = \dfrac{x}{(x^2+1)(x+i)(x+2)}$

$$\frac{(Ax+B)(x+1)(x+2)+C(x^2+1)(x+2)+D(x^2+1)(x+1)}{(x^2+1)(x+1)(x+2)} = \frac{x}{(x^2+1)(x+1)(x+2)}$$

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$$\frac{(Ax^2 + Ax + Bx + B)(x+2) + C(x^3 + 2x^2 + x + 2) + D(x^3 + x^2 + x + 1)}{(x^2+1)(x+1)(x+2)} = \frac{x}{(x^2+1)(x+1)(x+2)}$$

$$\frac{(Ax^3 + 2Ax^2 + Ax^2 + 2Ax + Bx^2 + 2Bx + Bx + 2B) + C(\cdots\cdots) + D(\cdots)}{(x^2+1)(x+1)(x+2)}$$

$$\frac{Ax^3 + 2Ax^2 + Ax^2 + 2Ax + Bx^2 + 2Bx + Bx + 2B + Cx^3 + 2Cx^2 + Cx + 2C + Dx^3 + Dx^2 + Dx + D}{(x^2+1)(x+1)(x+2)}$$

$$= \frac{x^3(A+C+D) + x^2(3A+B+2C+D) + x(2A+3B+C+D) + (2B+2C+D)}{(x^2+1)(x+1)(x+2)} = \frac{x}{(x^2+1)(x+1)(x+2)}$$

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$$\begin{cases} -3C - 3D - C - \frac{D}{2} + 2C + D = 0 \\ -2C - 2D - 3C - \frac{3}{2}D + C + D = 1 \end{cases}$$

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$$\begin{cases} 4C + 4D + D = 0 \\ -4C - D - \frac{3}{2}D = 1 \end{cases}$$

$$\Rightarrow \left(\cancel{-4C} - D - \frac{3}{2}D\right) + \left(\cancel{4C} + 4D + D\right) = 0+1$$

$$4D - \frac{3}{2}D = 1 \quad \Rightarrow \quad \frac{8D - 3D}{2} = 1$$

$$\Rightarrow 5D = 2$$

$$\Rightarrow \boxed{D = \frac{2}{5}}$$

$$\begin{cases} -2C - 2D - \frac{D}{2} = 0 \\ \boxed{D = \frac{2}{5}} \end{cases} \Rightarrow -2C - 2\cdot\frac{2}{5} - \frac{\frac{2}{5}}{2} = 0$$

$$\frac{2}{5}\cdot\frac{1}{2} = \frac{1}{5}$$

$$= -1 = \frac{-5}{5}$$

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$$\frac{Ax+B}{x^2+1} + \frac{C}{x+1} + \frac{D}{x+2} = \frac{x}{(x^2+1)(x+1)(x+2)}$$

Worksheet 4 :

Exercise 1 : Write out the general form for the partial fraction decomposition.

c) $\dfrac{x}{(x^2+1)(x+1)(x+2)} =$

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$= \dfrac{A\cancel{x}+B}{x^2+1} + \dfrac{C}{x+1} + \dfrac{D}{x+2} = \dfrac{x}{(x^2+1)(x+1)(x+2)}$

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$$\frac{(Ax^3+2Ax^2+Ax^2+2Ax+Bx^2+2Bx+Bx+2B)+C(\text{- - - - -})+D(\dots)}{(x^2+1)(x+1)(x+2)}$$

$$\frac{Ax^3 + 2Ax^2 + Ax^2 + 2Ax + Bx^2 + 2Bx + Bx + 2B + Cx^3 + 2Cx^2 + Cx + 2C + Dx^3 + Dx^2 + Dx + D}{(x^2+1)(x+1)(x+2)}$$

$$= \frac{x^3(A+C+D) + x^2(3A+B+2C+D) + x(2A+3B+C+D) + (2B+2C+D)}{(x^2+1)(x+1)(x+2)} = \frac{x}{(x^2+1)(x+1)(x+2)}$$

$$\begin{cases} A+C+D=0 \\ 3A+B+2C+D=0 \\ 2A+3B+C+D=1 \\ 2B+2C+D=0 \end{cases}$$

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$$\begin{cases} -3C - 3D - C - \frac{D}{2} + 2C + D = 0 \\ -2C - 2D - 3C - \frac{3}{2}D + C + D = 1 \end{cases} \Leftrightarrow \begin{cases} -2C - 2D - \frac{D}{2} = 0 \quad \cdot 2 \\ -4C - D - \frac{3}{2}D = 1 \end{cases}$$

$$\begin{cases} 4C + 4D + D = 0 \\ -4C - D - \frac{3}{2}D = 1 \end{cases}$$

$$\Rightarrow \left(-\cancel{4C} - D - \frac{3}{2}D\right) + \left(\cancel{4C} + 4D + D\right) = 0+1$$

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$$-2\cdot -\frac{1}{2} = 1$$

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$$B = \frac{3}{5}\cdot\frac{1}{2} = \boxed{\frac{3}{10} = B}$$

$$\frac{Ax+B}{x^2+1} + \frac{C}{x+1} + \frac{D}{x+2} = \frac{x}{(x^2+1)(x+1)(x+2)}$$

$$\frac{\frac{1}{10}x + \frac{3}{10}}{x^2+1} + \frac{\frac{2}{5}}{x+1} + \frac{-\frac{1}{2}}{x+2}$$

Exercise 1 :

a) $\frac{1}{x^2+3x+2}$

$= \frac{1}{(x-(-1))(x-(-2))}$

$= \frac{1}{(x+1)(x+2)} = \frac{A}{x+1} + \frac{B}{x+2}$

$x^2+3x+2=0$

$x_{1,2} = \frac{-b \pm \sqrt{b^2-4a \cdot c}}{2 \cdot a}$

Using Quadratic formula

$ax^2+bx+c=0$

$x_{1,2} = \frac{-3 \pm \sqrt{9-4 \cdot 2}}{2} =$

$= \frac{-3 \pm \sqrt{1}}{2} = \frac{-3 \pm 1}{2}$

$x_1 = \frac{-3+1}{2}$, $x_2 = \frac{-3-1}{2}$

$x_1 = -1$ $\qquad$ $x_2 = -2.$