Notes :

$\sin^2 x + \cos^2 x = 1$

$$\frac{x}{\sqrt{4-x^2}} \to \underbrace{4}_{2^2} - x^2 \to x = 2\sin t$$

$$4 - (2\sin t)^2 = 4 - 4\sin^2 t$$

$$= 4(\underbrace{1 - \sin^2 t}_{}) $$

$$4 \cdot \cos^2 t$$

Notes:

$$\frac{x^3}{\sqrt{1+x^2}}$$

?

$$x = 1 \cdot tg(t) = \frac{\sin(t)}{\cos(t)}$$

$$1+x^2 = 1 + \frac{\sin^2(t)}{\cos^2(t)} = \frac{\cos^2(t)}{\cos^2(t)} + \frac{\sin^2(t)}{\cos^2(t)} =$$

$$= \frac{\cos^2(t) + \sin^2(t)}{\cos^2(t)} = \frac{1}{\cos^2(t)}$$

$$\frac{\cancel{\cos^2(t)}}{\cancel{\cos^2(t)}} = 1$$

$$\sin^2(t) + \cos^2(t) = 1 \Rightarrow \cos^2(t) = 1 - \sin^2(t)$$

Exercise 1:

Use the trigonometric substitution $x = \sin(t)$ to find $\int \frac{1}{\sqrt{1-x^2}}\,dx = I$

$$\boxed{x = \sin(t)} \Rightarrow dx = d(\sin(t)) = (\sin(t))' \cdot dt = \cos(t) \cdot dt$$

$$I = \int \frac{1}{\sqrt{\underbrace{1 - \sin^2(t)}_{= \cos^2(t)}}} \cdot \cos t \cdot dt = \int \frac{\cos(t) \cdot dt}{\sqrt{\cos^2(t)}} = \int \frac{\cancel{\cos(t)}}{\cancel{\cos(t)}}\,dt = \int dt =$$

$I = \int dt = \boxed{t + c} = \boxed{\arcsin(x) + c = I}$

$\boxed{x = \sin(t)}$ → inverse function of sinus at both sides $\Rightarrow \arcsin(\cdot)$

$\boxed{\arcsin(x) = \arcsin(\sin(t)) = t}$

$x = 2$

apply the function

Notes:

$$\frac{x}{\sqrt{4-x^2}}$$

$\uparrow$

$\rightarrow \quad \underbrace{4}_{\overset{\shortparallel}{2^2}} - x^2 \rightarrow \quad x = 2\sin t$

$\sin^2 x + \cos^2 x = 1$

$$4 - (2\sin t)^2 = 4 - 4\sin^2 t$$

$$= 4(1 - \sin^2 t)$$

$\shortparallel$

$4 \cdot \cos^2 t$

Notes:

$$\frac{x^3}{\sqrt{1+x^2}}$$

?

$$x = 1 \cdot tg(t) = \frac{\sin(t)}{\cos(t)}$$

$$1+x^2 = 1 + \frac{\sin^2(t)}{\cos^2(t)} = \frac{\cos^2(t)}{\cos^2(t)} + \frac{\sin^2(t)}{\cos^2(t)} =$$

$$= \frac{\cos^2(t) + \sin^2(t)}{\cos^2(t)} = \frac{1}{\cos^2(t)}$$

$$\frac{\cancel{\cos^2(t)}}{\cancel{\cos^2(t)}} = 1$$

$$\int_{\pi/2}^{\pi/6} \frac{(4\sin(t))^3}{\sqrt{16-16\sin^2(t)}} \cdot 4\cos(t)\,dt = \int_{\pi/2}^{\pi/6} \frac{4^3 \cdot \sin^3(t)}{\sqrt{16(\underbrace{1-\sin^2(t)}_{\cos^2(t)})}} \cdot 4\cos(t)\,dt =$$

$$= 4^3 \int_{\pi/2}^{\pi/6} \frac{\sin^3(t)}{\sqrt{16}\cdot\sqrt{\cos^2(t)}} \cdot 4\cos(t)\,dt = 4^3 \int_{\pi/2}^{\pi/6} \frac{\sin^3(t)}{\cancel{4\cdot\cos(t)}} \cdot \cancel{4\cos(t)} \cdot dt =$$

$$= 4^3 \int_{\pi/2}^{\pi/6} \sin^3(t)\,dt = 4^3 \int_{\pi/2}^{\pi/6} \sin^2(t)\cdot \sin(t)\,dt$$

substitute

$x = \cos(t) \rightarrow$

$dx = -\sin(t)\,dt \Rightarrow -dx = \sin(t)\,dt$

$$I = 4^3 \int_{\pi/2}^{\pi/6} \left(1-\cos^2(t)\right) \cdot \sin t \, dt$$

$-dx$

$$x = \cos(t)$$

$$t = \frac{\pi}{2} \to x = \cos\frac{\pi}{2} = 0$$

$$t = \frac{\pi}{6} \to x = \cos\frac{\pi}{6} = \frac{\sqrt{2}}{2}$$

$$I = 4^3 \int_{0}^{\frac{\sqrt{2}}{2}} (1-x^2) \cdot -dx = -4^3 \int_{0}^{\frac{\sqrt{2}}{2}} (1-x^2)\, dx =$$

$$= -4^3 \left(\int_0^{\frac{\sqrt{2}}{2}} 1 \cdot dx - \int_0^{\frac{\sqrt{2}}{2}} x^2 \, dx \right) = -4^3 \left(x \Big|_0^{\frac{\sqrt{2}}{2}} - \frac{x^3}{3} \Big|_0^{\frac{\sqrt{2}}{2}} \right)$$

$$\sin^2(t) = 1 - \cos^2(t)$$

$$x = \cos(t)$$

$$-d(\cos(t))$$

$$I = 4^3 \int_{\pi/6}^{\pi/2} \sin^2(t)\,\sin(t)\,dt = 4^3 \int_{\pi/6}^{\pi/2} \left(1 - \cos^2(t)\right) \sin(t)\,dt$$

$$I = -4^3 \left(x \Big|_0^{\frac{\sqrt{2}}{2}} - \frac{x^3}{3} \Big|_0^{\frac{\sqrt{2}}{2}} \right) =$$

$$= -4^3 \left(\left(\frac{\sqrt{2}}{2} - 0 \right) - \left(\frac{\left(\frac{\sqrt{2}}{2}\right)^3}{3} - \frac{0^3}{3} \right) \right)$$

Notes:

$$\boxed{\sin^2(t) + \cos^2(t) = 1}$$

$$\int \frac{x}{\sqrt{4 - x^2}}\,dx$$

$\sqrt{4}$

$$x = 2\sin(t) \Rightarrow dx = d(2\sin(t))$$

$$= 2\cos(t)\,dt$$

$$4 - 4\sin^2(t) = 4(1 - \sin^2(t)) = 4\cdot\cos^2(t)$$

$$\int \frac{x}{\sqrt{16 + x^2}}\,dx$$

$$x = 4\,tg(t)$$

$\sqrt{16}$

$$\boxed{tg(t) = \frac{\sin(t)}{\cos(t)}}$$

$$16 + 16\,tg^2(t) = 16(1 + tg^2(t)) = 16\left(1 + \frac{\sin^2(t)}{\cos^2(t)}\right)$$

$$= 16\left(\frac{\cos^2(t)}{\cos^2(t)} + \frac{\sin^2(t)}{\cos^2(t)}\right) = 16\left(\frac{\cos^2(t) + \sin^2(t)}{\cos^2(t)}\right) = 16\cdot\frac{1}{\cos^2(t)}$$

$$\sin^2(u) + \cos^2(u) = 1 \to 1 - \sin^2(u) = \cos^2(u)$$

Exercise 1: Use the trigonometric substitution $x = \sin(u)$ to find $\int \frac{1}{\sqrt{1-x^2}}\, dx = I$

let $\boxed{x = \sin(u)} \Rightarrow dx = d(\sin(u)) = (\sin(u))' \, du = \boxed{\cos(u)\, du}$

$$I = \int \frac{1}{\sqrt{1-\sin^2(u)}} \cdot \cos(u)\, du = \int \frac{\cos(u)\, du}{\sqrt{\cos^2(u)}} = \int \frac{\cancel{\cos(u)}\, du}{\cancel{\cos(u)}} =$$

$$= \int du = \boxed{u + C} = I$$

$\boxed{x = \sin(u)}$ → take the $\underbrace{\text{inverse of}}_{\arcsin(\cdot)}$ sinus at both sides

$$\arcsin(x) = \arcsin(\sin(u)) = \boxed{u = \arcsin(x)}$$

$$I = u + c = \boxed{\arcsin(x) + c = I}$$

Exercise 2 :

a) $\displaystyle\int_0^2 \frac{u^3}{\sqrt{16-u^2}}\,du$

$\boxed{u = 4\sin(x)}$

$\sqrt{16}$ $\Rightarrow$ $du = d(4\sin(x))$

$= (4\sin(x))'\,dx = \boxed{4\cos(x)\,dx}$

$\frac{u}{4} = \sin(x) \Rightarrow x = \arcsin\left(\frac{u}{4}\right)$

$u = 0 \rightarrow x = \arcsin\left(\frac{0}{4}\right) = 0$

$u = 2 \rightarrow x = \arcsin\left(\frac{2}{4}\right) = \frac{\pi}{6}$

$$\int_0^{\frac{\pi}{6}} \frac{(4\sin(x))^3}{\sqrt{16-16\sin^2(x)}} \cdot 4\cos(x)\,dx = \int_0^{\frac{\pi}{6}} \frac{4^3 \sin^3(x)}{\sqrt{16\cdot(\underbrace{1-\sin^2(x)}_{\cos^2(x)})}} \cdot 4\cos(x)\,dx =$$

$$= \int_0^{\frac{\pi}{6}} \frac{4^3 \cdot \sin^3(x)}{\sqrt{16\cdot\cos^2(x)}} \cdot 4\cos(x)\,dx = \int_0^{\frac{\pi}{6}} \frac{4^3 \sin^3(x)}{\cancel{4\cos(x)}} \cdot \cancel{4\cos(x)}\,dx =$$

$$= 4^3 \int_0^{\frac{\pi}{6}} \sin^3(x)\,dx$$

$$\boxed{\begin{aligned} \sin^2(x) + \cos^2(x) &= 1 \\ 1-\sin^2(x) &= \cos^2(x) \end{aligned}}$$

$$I = 4^3 \int_0^{\frac{\pi}{6}} \sin^3(x)\,dx =$$

$$t = \cos(x)$$

$$-dt = \sin(x)\,dx$$

$$dt = -\sin(x)\,dx$$

$$= 4^3 \int_0^{\frac{\pi}{6}} \underbrace{\sin^2(x)}\,\sin(x)\,dx$$

$x = 0 \to t = \cos(0) = 1$

$x = \frac{\pi}{6} \to t = \cos\left(\frac{\pi}{6}\right) = \frac{\sqrt{2}}{2}$

$$= 4^3 \int_0^{\frac{\pi}{6}} \left(1 - \cos^2(x)\right) \cdot \sin(x)\,dx = 4^3 \int_1^{\frac{\sqrt{2}}{2}} \left(1 - t^2\right) \cdot (-dt) =$$

$$I = 4^3 \int_{\frac{\sqrt{2}}{2}}^{1} (1 - t^2)\, dt = 4^3 \left(\int_{\frac{\sqrt{2}}{2}}^{1} 1 \cdot dt - \int_{\frac{\sqrt{2}}{2}}^{1} t^2\, dt \right)$$

$$= 4^3 \left(t \Big|_{\frac{\sqrt{2}}{2}}^{1} - \frac{t^3}{3} \Big|_{\frac{\sqrt{2}}{2}}^{1} \right) = 4^3 \underbrace{\left(\left(1 - \frac{\sqrt{2}}{2}\right) - \left(\frac{1}{3} - \frac{\left(\frac{\sqrt{2}}{2}\right)^3}{3} \right) \right)}$$