a) $\int x \cos(x^2)\, dx$

$du = dx^2 = (x^2)' \cdot dx$

$= 2x\, dx$

$u = x^2$

$= \int \frac{2}{2} x \cos(x^2)\, dx =$

$= \frac{1}{2} \int \cos(x^2)\, 2x\, dx = \frac{1}{2} \int \cos(x^2)\, dx^2 = \frac{1}{2} \int \cos u\, du =$

$= \frac{1}{2} \sin u + C$

b) $\int e^x \sin x\, dx$ → integration by part

dv

$\boxed{u = e^x}$ → $du = e^x dx$

$\boxed{dv = \sin x\, dx}$ → $v = \int dv = \int \sin x\, dx = -\cos x$

In general

$\int u\, dv = u \cdot v - \int v\, du$

$$\overbrace{\boxed{\int e^x \sin x \, dx}}^{I} = e^x \cdot (-\cos x) - \int (-\cos x) \cdot e^x \, dx =$$

$$= -e^x \cos x + \boxed{\int \cos x \, e^x \, dx} = J$$

$$u = e^x \longrightarrow du = e^x \, dx$$

$$dv = \cos x \, dx \rightarrow v = \int \cos x \, dx = \sin x$$

$$\int u \, dv = u \cdot v - \int v \, du$$

$$J = e^x \cdot \sin x - \int \sin x \, e^x \, dx$$

$$I = -e^x \cos x + e^x \sin x - I$$

$$\Rightarrow 2I = -e^x \cos x + e^x \sin x$$

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$= 2x\,dx \Rightarrow$ $\boxed{\frac{du}{2} = x\,dx}$

$\int \cos x\,dx = \sin x$

$\int \cos x^2\, x\,dx = \int \cos u \cdot \frac{du}{2}$

$= \frac{1}{2} \int \cos u\,du = \frac{1}{2} \sin u + C$

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$= \frac{1}{2} \sin x^2 + C$

$$\int \underbrace{4x}\,\underbrace{\sin x}\,dx$$

$$\int \underbrace{x}\,\cos(x^2)\,dx$$

$u = 4x \rightsquigarrow du = 4\,dx$

$dv = \sin x\,dx \longrightarrow v = \int dv = \int \sin x\,dx$

Exercise 2:

a) $\int x^2 \sin x \, dx$ $\to x$

$(x^2)'$

$U = x^2 \longrightarrow du = 2x \, dx$

$dv = \sin x \, dx \longrightarrow V = \int dv = \int \sin x \, dx = -\cos x$

$$\boxed{\int u \, dv = u \cdot V - \int V \, du}$$

a) $\int x \cos(x^2)\, dx$

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$= \frac{1}{2} \sin x^2 + C$

$\int 4x \sin x \, dx$

$\int x \cos x^2 \, dx$

$u = 4x \rightsquigarrow du = 4\,dx$

$dv = \sin x \, dx \longrightarrow v = \int dv = \int \sin x \, dx$

Exercise 2:

a) $\int x^2 \sin x \, dx$

$(x^2)'$

$\boxed{\int u\,dv = u \cdot v - \int v\,du}$

$\boxed{u = x^2} \longrightarrow du = 2x\,dx$ $\quad \boxed{du = 2x\,dx}$

$\boxed{dv = \sin x\,dx} \longrightarrow v = \int dv = \int \sin x\,dx = \boxed{-\cos x = v}$

$\int x^2 \sin x\,dx = x^2 \cdot (-\cos x) - \int -\cos x \cdot 2x\,dx =$

$= -x^2 \cos x + \boxed{\int 2x \cos x\,dx} = I$

$$I = \int 2x \cos x \, dx$$

$$\int u\,dv = u \cdot v - \int v\,du$$

$$\boxed{u = 2x} \rightarrow du = (2x)' \, dx = \boxed{2\,dx = du}$$

$$\boxed{dv = \cos x \, dx} \rightarrow v = \int dv = \int \cos x \, dx = \boxed{\sin x = v}$$

$$I = 2x \cdot \sin x - \int \sin x \cdot 2\,dx = 2x \sin x - 2\int \sin x \, dx$$

$$= 2x \sin x - 2(-\cos x) + C$$

Answer: $\int x^2 \sin x \, dx = -x^2 \cos x + \int 2x \cos x \, dx = -x^2 \cos x + (2x \sin x + 2\cos x + C)$

Exercise 1 :

c) $\int \frac{\ln(\arctan(x))}{1+x^2}\,dx$

$= \frac{1}{1+x^2}\,dx$

$u = \arctan(x)$

$I = \int \ln u \, du$

$$I = \int \frac{\ln(\arctan(x))}{1+x^2}\,dx$$

$$\boxed{u = \arctan(x)}$$

$$du = d(\arctan(x)) = (\arctan(x))' \cdot dx = \frac{1}{1+x^2} \cdot dx$$

$$\boxed{du = \frac{1}{1+x^2} \cdot dx}$$

$$I = \int \ln u \cdot 1 \cdot du$$

$$k = \ln u \rightarrow dk = (\ln u)'\, du = \frac{1}{u}\, du$$

$$dl = 1 \cdot du \rightarrow l = \int dl = \int du = u$$

$$I = \ln u \cdot u - \int u \cdot \frac{du}{u} = \ln u \cdot u - \int du = \ln u \cdot u - u + c =$$

$$= \ln(\arctan(x)) \cdot \arctan(x) - \arctan(x) + C$$

$$I = \ln(u) \cdot u - u + C =$$

$$= \boxed{\ln(\arctan(x)) \cdot \arctan(x) - \arctan(x) + C = I}$$

Exercise 1 :

d) $\int x e^{x^2} dx$ — $\underline{u = x^2}$

$\frac{du}{2} = (x^2)' dx = \frac{2x\,dx}{2}$

$\boxed{\frac{du}{2} = x\,dx}$

$\int \underbrace{x e^{x^2} dx} = \int e^u \frac{du}{2} = \frac{1}{2} \int e^u du = \frac{1}{2} e^u + c$

$= \boxed{\frac{1}{2} e^{x^2} + c}$