Worksheet 9:

Exercise 2:

a) For what values of x does the sequence $\{x^n\}$ converge?

We need to check $\lim_{n\to\infty} x^n$

We need to see some cases by first taking numbers:

1) if $-1 \leq x \leq 1$, we can take $x = \frac{1}{2}$.

$\lim_{n\to\infty} \left(\frac{1}{2}\right)^n = 0$ Because we can see that 2^n goes to infinity faster than $1^n = 1$

In general, if we have $-1 \leq x \leq 1$ we can use

2) if $x > 1$, let $x = 3$, $\lim_{n\to\infty} 3^n = \infty$. So the sequence diverges.

b) For what values of x does the sequence $\{n^x\}$ converge?

We need to check $\lim_{n\to\infty} n^x$. We need to see some cases by first taking some numbers and then look at it in general!

if $x < 0$, let $x = -2$, $\lim_{n\to\infty} n^{-2} = \lim_{n\to\infty} \frac{1}{n^2} = 0$

This also works in general since every time we have

$$\lim_{n\to\infty} n^{-a} = \lim_{n\to\infty} \frac{1}{n^a} = 0.$$

a is positive

if $x>0$, let $x=3$, $\lim_{n\to\infty} n^3 = \infty$ and this is true for every $x>0$.

c) if $\lim_{n\to\infty} b_n = \sqrt{2}$ find $\lim_{n\to\infty} b_{n-3} = ?$

$\lim_{n\to\infty} b_{n-3} = \sqrt{2}$ because if we add or subtract a finite number of elements the limit does not change.

Exercise 5 :

Suppose the sequence $a_1, a_2, a_3, \dots$ satisfies the recursion relation

$$\boxed{a_n = \frac{1}{2}\left(a_{n-1} + \frac{3}{a_{n-1}}\right),} \quad n \geq 2.$$

and $a = \lim_{n\to\infty} a_n$ exists. find a?

Since $\lim_{n\to\infty} a_n = a$ exists this means that our sequence converges

$a \neq 0$

So from here we get..

$$a = \frac{1}{2}\left(a + \frac{3}{a}\right), \quad a = \frac{1}{2}a + \frac{3}{2a}$$

$$2a^2 = a^2 + 3, \quad a^2 = 3,$$

$$a = \pm\sqrt{3}.$$

Since we have that limit exists, it should be either $\sqrt{3}$ or $-\sqrt{3}$ (Be carful we are not looking if the limit exists or not.