Worksheet 28:

Exercise 5:

$\Delta x = 0,5$,
$$\begin{cases} y' = y - 2x \\ y(1) = 0 \end{cases}$$

By using the formula we get $y_{n+1} = y_n + \Delta x \cdot f(x_n, y_n)$

$$= y_n + 0,5 \cdot (y_n - 2x_n)$$

$$= y_n(1 + 0,5) - y_n =$$

$$= 1,5 \cdot y_n - x_n$$

Since the step size $\Delta x = 0,5 \Rightarrow x_0 = 1$, $x_1 = 1,5$, $x_2 = 2$.
$x_3 = 2,5$, $x_4 = 3$.

Then we have $y_0 = 0 \Rightarrow$

$y_1 = 1,5 \cdot y_0 - x_0 = 0 - 1 = -1$

$y_2 = 1,5 \cdot y_1 - x_1 = -1,5 - 1,5 = -3$

$y_3 = 1,5\, y_2 - x_2 = -4,5 - 2,5 = -7$

$y_4 = 1,5 \cdot y_3 - x_3 = -10,5 - 3 = -13,5$

Exercise 6:

(a) $y' + 4xy^2 = 0$

$4xy^2 = -y'$

$4x = \frac{-1}{y^2}\frac{dy}{dx}$

$\int 4x\,dx = \int \frac{-1}{y^2}\,dy$

$2x^2 + C = \frac{1}{y}$

$\boxed{y = \frac{1}{2x^2+C}}$

(b) $\sqrt{1-x^2}\,\frac{dy}{dx} = x^3 y$

$\int \frac{1}{y}\,dy = \int \frac{x}{\sqrt{1-x^2}}\,dx$

Let
$u = 1-x^2$
$du = -2x\,dx$
$-\frac{1}{2}du = x\,dx$

$\ln|y| = -\frac{1}{2}\int \frac{1}{\sqrt{u}}\,du$

$\ln|y| = -\frac{1}{2}\left(2\sqrt{1-x^2}\right) + C = -\sqrt{1-x^2} + C$

$y = e^{-\sqrt{1-x^2}+C} = C \cdot e^{-\sqrt{1-x^2}}$

c) $(1+x^2)\frac{dy}{dx} = x^3 y$

$\int \frac{dy}{y} = \int \frac{x^3}{1+x^2}\,dx$

$$\begin{array}{r|l} x^3 & x^2+1 \\ \hline -(x^3+x) & x \\ -x & \end{array}$$

$\Rightarrow \frac{x^3}{x^2+1} = x - \frac{x}{x^2+1}$

$u = x^2+1 \Rightarrow du = 2x\,dx$

$\frac{1}{2}du = x\,dx$

$\ln|y| = \int \left(x - \frac{x}{x^2+1}\right)dx = \frac{x^2}{2} - \frac{1}{2}\ln|x^2+1| + C$

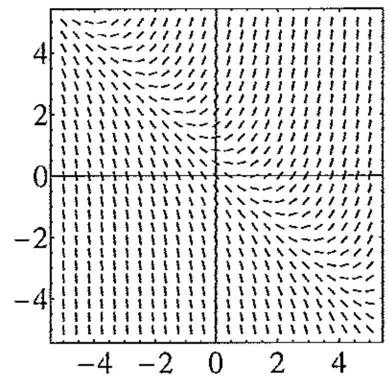
$\boxed{y = e^{\left(\frac{x^2}{2} - \frac{1}{2}\ln|x^2+1| + C\right)}}$

MA 114 Worksheet # 28: Graphical Methods

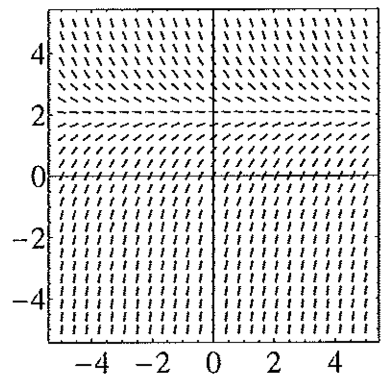
1. Match the differential equation with its slope field. Give reasons for your answer.

$$y' = 2 - y \qquad y' = x(2-y) \qquad y' = x + y - 1 \qquad y' = \sin(x)\sin(y)$$

$y' = x + y - 1$
Positive above the line $x+y-1=0$ and negative below.



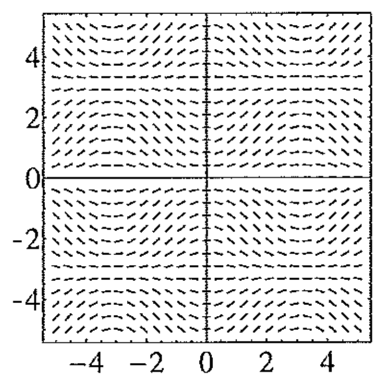
(a) Slope field I



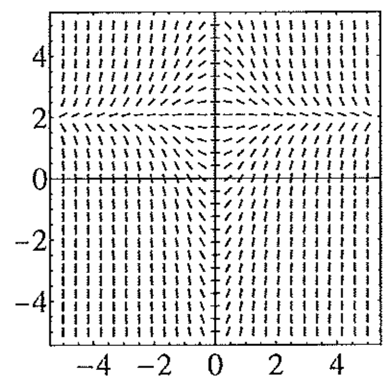
(b) Slope field II

$y' = 2 - y$
Zero when $y=2$, does not change with x.

$y' = \sin(x)\sin(y)$
Oscillatory behavior.



(c) Slope Field III



(d) Slope field IV

$y' = x(2-y)$
Zero when $x=0$ or $y=2$.

Figure 1: Slope fields for Problem 1

2. Use slope field labeled IV to sketch the graphs of the solutions that satisfy the given initial conditions

$$y(0) = -1, \quad y(0) = 0, \quad y(0) = 1.$$

3. Sketch the slope field of the differential equation. Then use it to sketch a solution curve that passes through the given point
   (a) $y' = y - 2x$, $(1, 0)$
   (b) $y' = xy - x^2$, $(0, 1)$

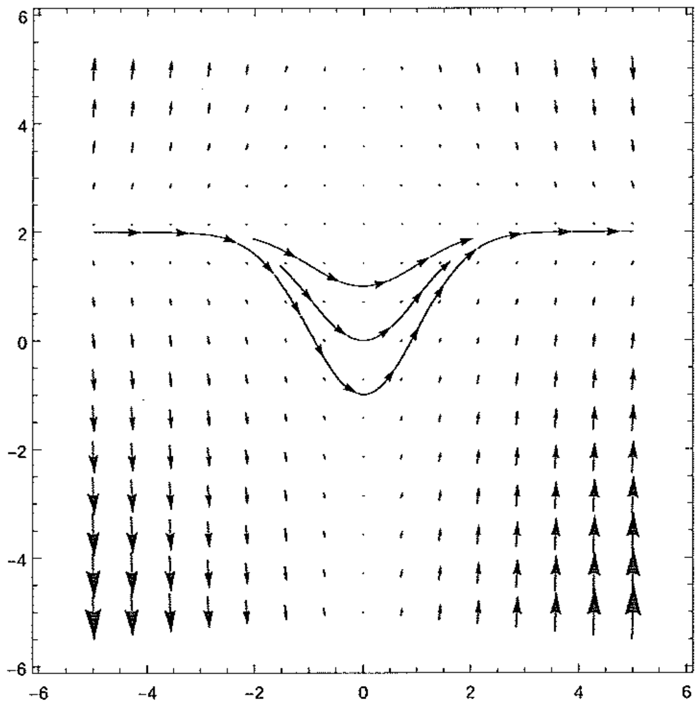
4. Show that the isoclines of $y' = t$ are vertical lines. Sketch the slope field for $-2 \le t \le 2$, $-2 \le y \le 2$ and plot the integral curves passing through $(0, 1)$ and $(0, -1)$.

Isoclines are regions with the same slope.
Integral curve is just a fancy word for "solution function".

2) In[19]:=

```
VectorPlot[{1, x (2 - y)}, {x, -5, 5}, {y, -5, 5},
  StreamPoints → {{{{0, -1}, Black}, {{0, 0}, Black}, {{0, 1}, Black}}}]
```

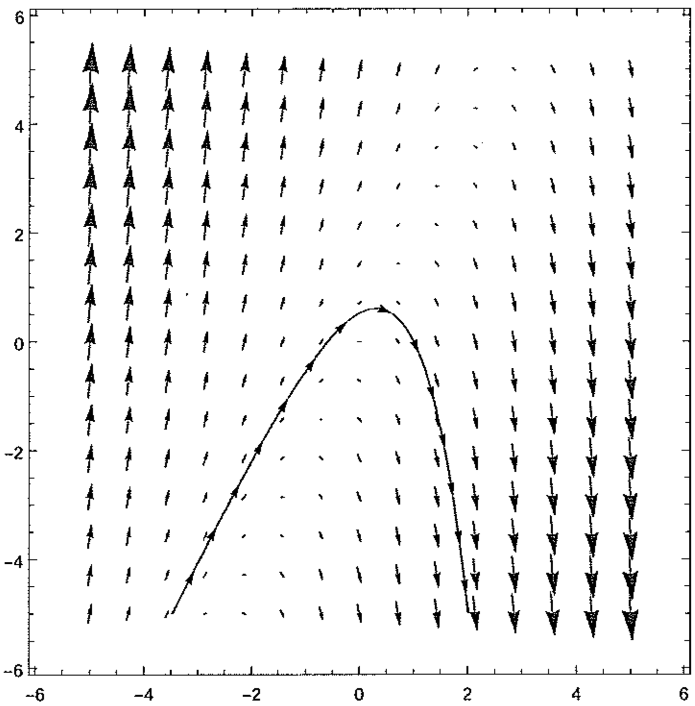
Out[19]=



3a) In[12]:=

```
VectorPlot[{1, y - 2 x}, {x, -5, 5}, {y, -5, 5}, StreamPoints → {{{{1, 0}, Black}}}]
```

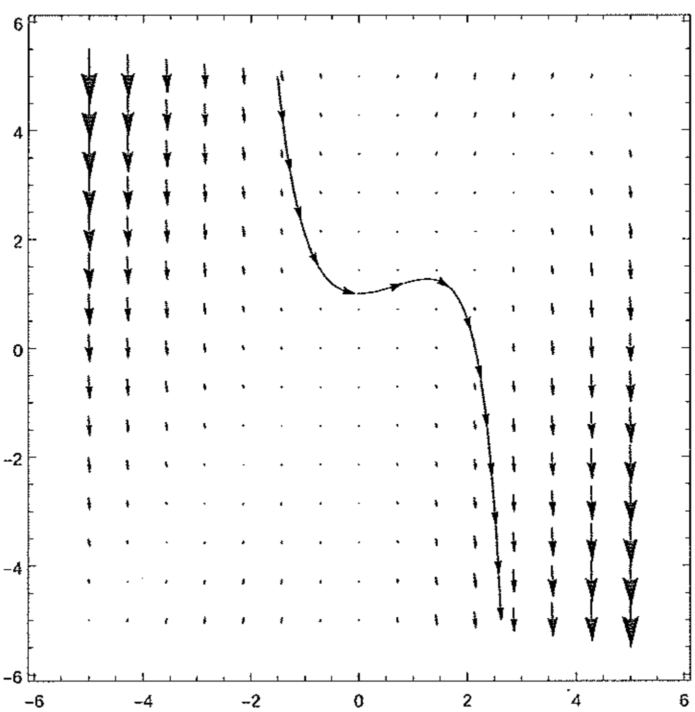
Out[12]=



b) In[13]:=

```
VectorPlot[{1, x y - x^2}, {x, -5, 5}, {y, -5, 5}, StreamPoints → {{{{0, 1}, Black}}}]
```

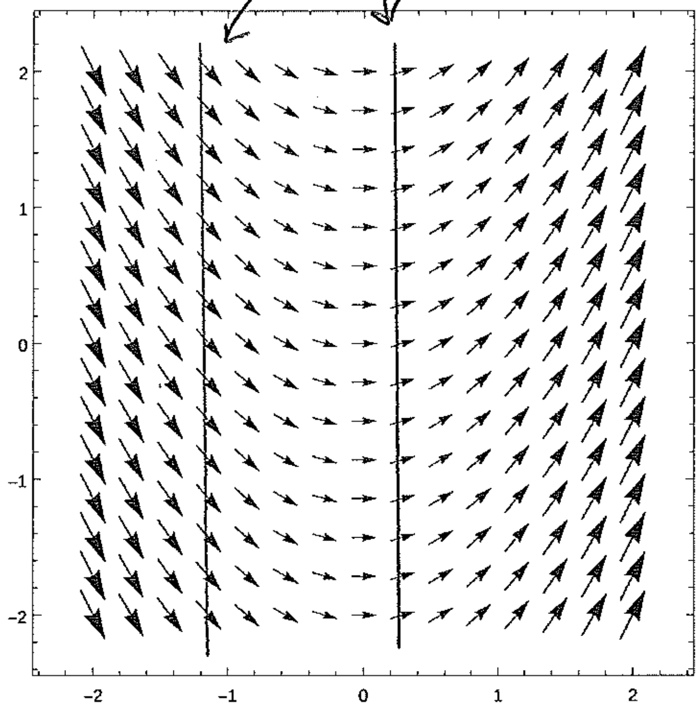
Out[13]=



isoclines

4) In[18]:= `VectorPlot[{1, t}, {t, -2, 2}, {y, -2, 2}]`

Out[18]=



In[17]:= `VectorPlot[{1, t}, {t, -2, 2}, {y, -2, 2}, StreamPoints → {{{{0, 1}, Black}, {{0, -1}, Black}}}]`

Out[17]=