Worksheet 23,

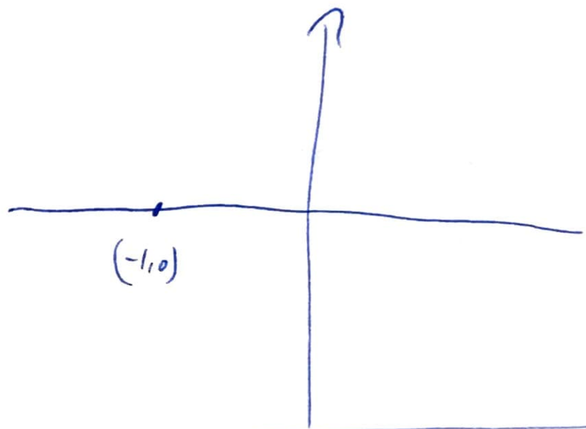
Exercise 1,

(a) $(1, \sqrt{3})$; $r = \sqrt{1^2 + (\sqrt{3})^2} = \sqrt{1+3} = \sqrt{4} = 2$

$\tan(\theta) = \frac{y}{x} = \frac{\sqrt{3}}{1} = \sqrt{3} \Rightarrow \theta = \tan^{-1}(\sqrt{3}) = \frac{\pi}{3}$

Polar coordinates $(2, \frac{\pi}{3})$

(b) $(-1, 0)$; $r = \sqrt{(-1)^2 + 0^2} = \sqrt{1+0} = \sqrt{1} = 1$.

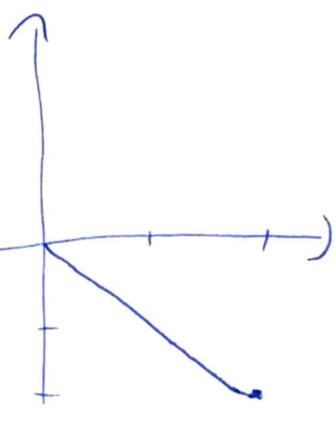


$\tan(\theta) = \frac{y}{x} = \frac{0}{-1} = 0$

$\theta = \pi \Rightarrow$ Polar coordinates $(1, \pi)$.

(c) $(2, -2)$; $r = \sqrt{2^2 + (-2)^2} = \sqrt{4+4} = \sqrt{8}$

$\boxed{r = 2\sqrt{2}}$



$\tan(\theta) = \frac{y}{x} = \frac{-2}{2} = -1$, $\theta = \tan^{-1}(-1) = \frac{-\pi}{4}$

Polar coordinates $(2\sqrt{2}, \frac{-\pi}{4})$

Exercise 2 ;

(a) $\left(2, \frac{\pi}{6}\right)$; $x = r\cos\theta = 2\cdot\cos\frac{\pi}{6} = 2\cdot\frac{\sqrt{3}}{2} = \sqrt{3}$

$y = r\sin\theta = 2\cdot\sin\frac{\pi}{6} = 2\cdot\frac{1}{2} = 1.$

Rectangular coordinates $(\sqrt{3}, 1)$.

(b) $\left(-1, \frac{\pi}{2}\right)$; $x = r\cos\theta = (-1)\cdot\cos\frac{\pi}{2} = -1\cdot 0 = 0$

$y = r\cdot\sin\theta = (-1)\cdot\sin\frac{\pi}{2} = -1\cdot 1 = -1$

Rectangular coordinates $(0, -1)$.

(c) $\left(1, \frac{\pi}{4}\right)$; $x = r\cos\theta = 1\cdot\cos\frac{\pi}{4} = 1\cdot\frac{\sqrt{2}}{2} = \frac{\sqrt{2}}{2}.$

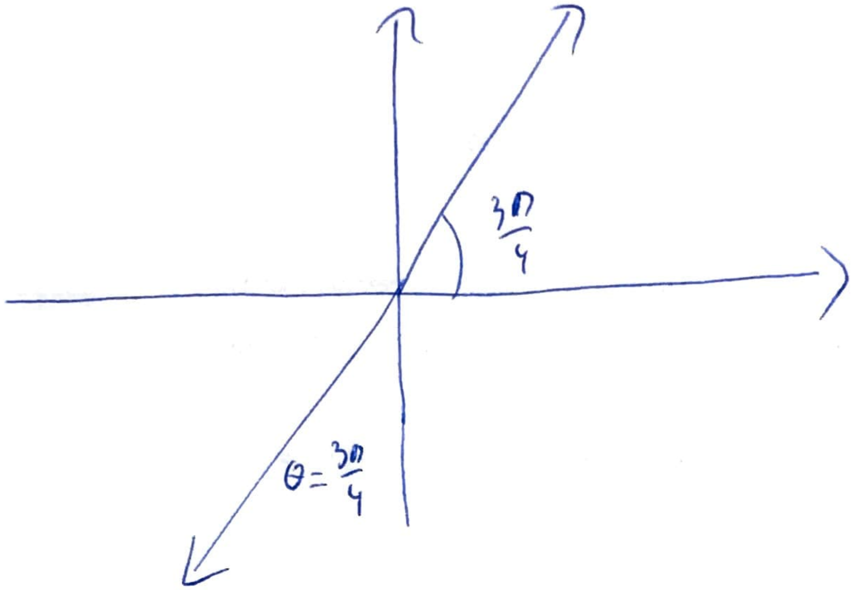
$y = r\sin\theta = 1\cdot\sin\frac{\pi}{4} = 1\cdot\frac{\sqrt{2}}{2} = \frac{\sqrt{2}}{2}$

Rectangular coordinates $\left(\frac{\sqrt{2}}{2}, \frac{\sqrt{2}}{2}\right)$.

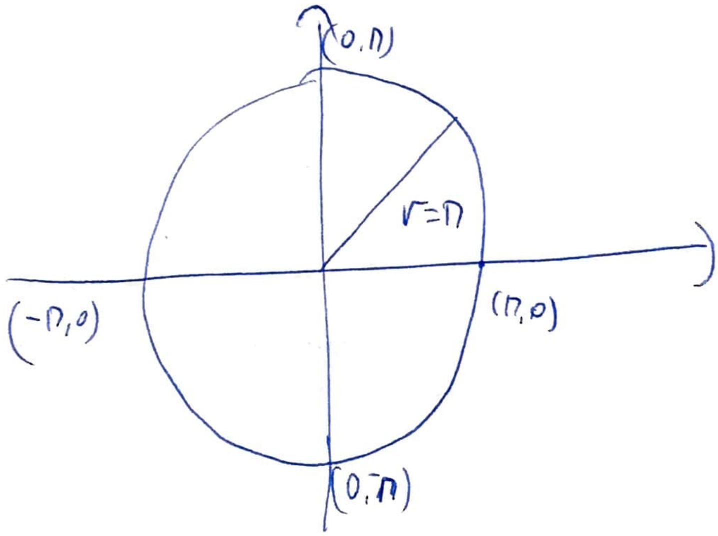
Exercise 3 ;

Exercise 4 :

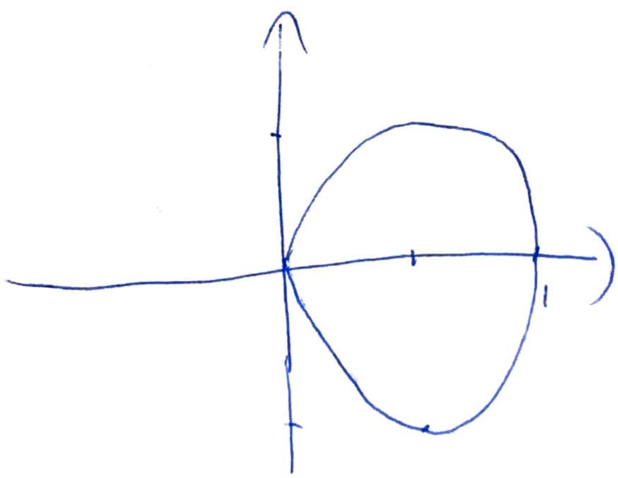
(a) $\theta = \frac{3\pi}{4}$



(b) $r = \pi$

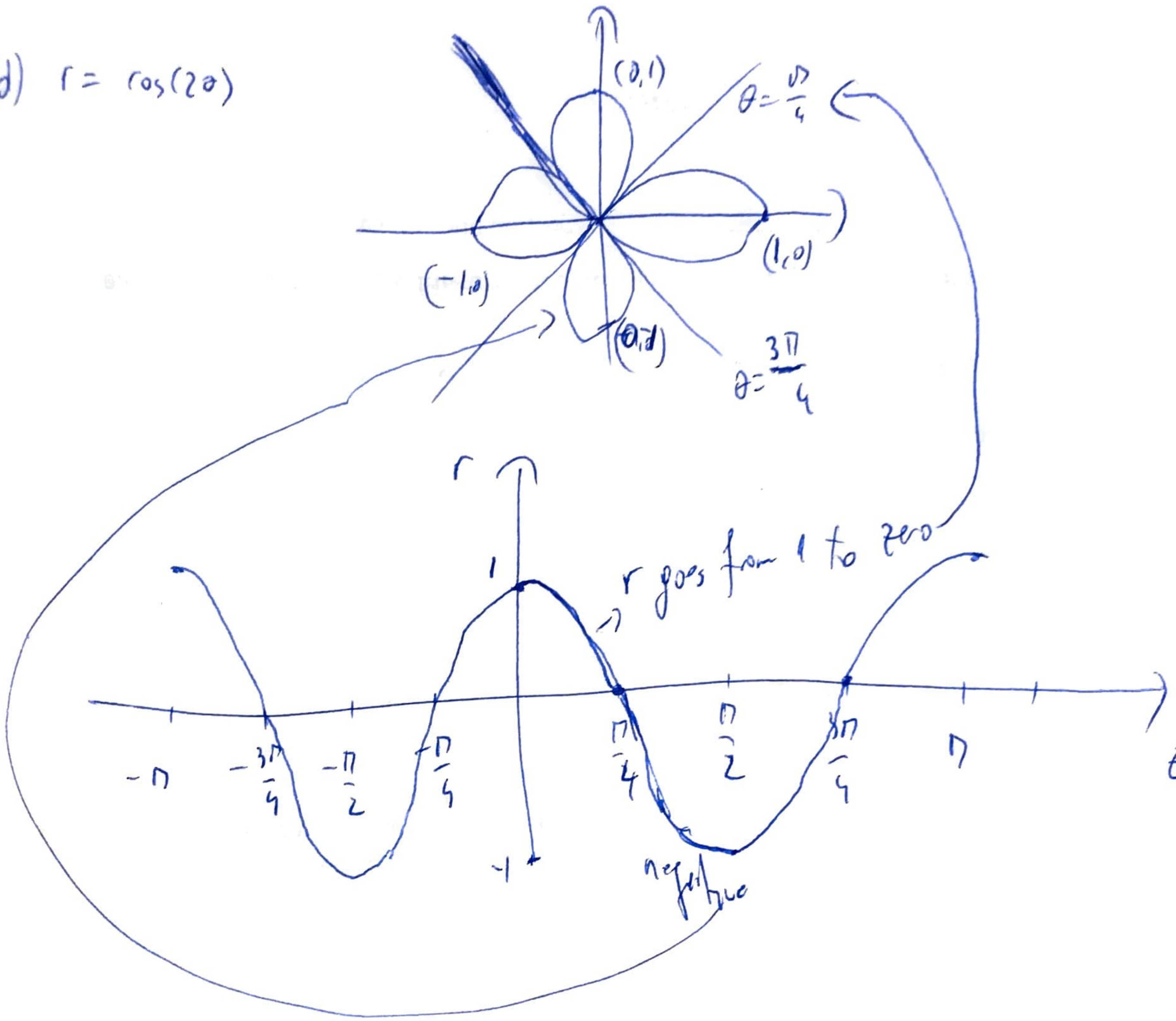


(c) $r = \cos(\theta)$

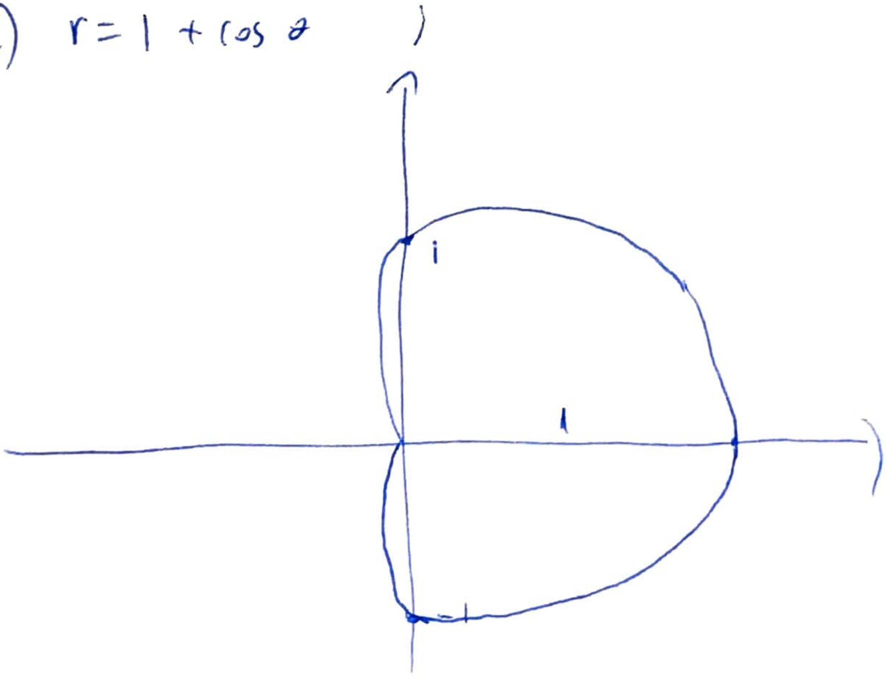


θ	r
0	1
$\frac{\pi}{2}$	0
π	-1

(d) $r = \cos(2\theta)$

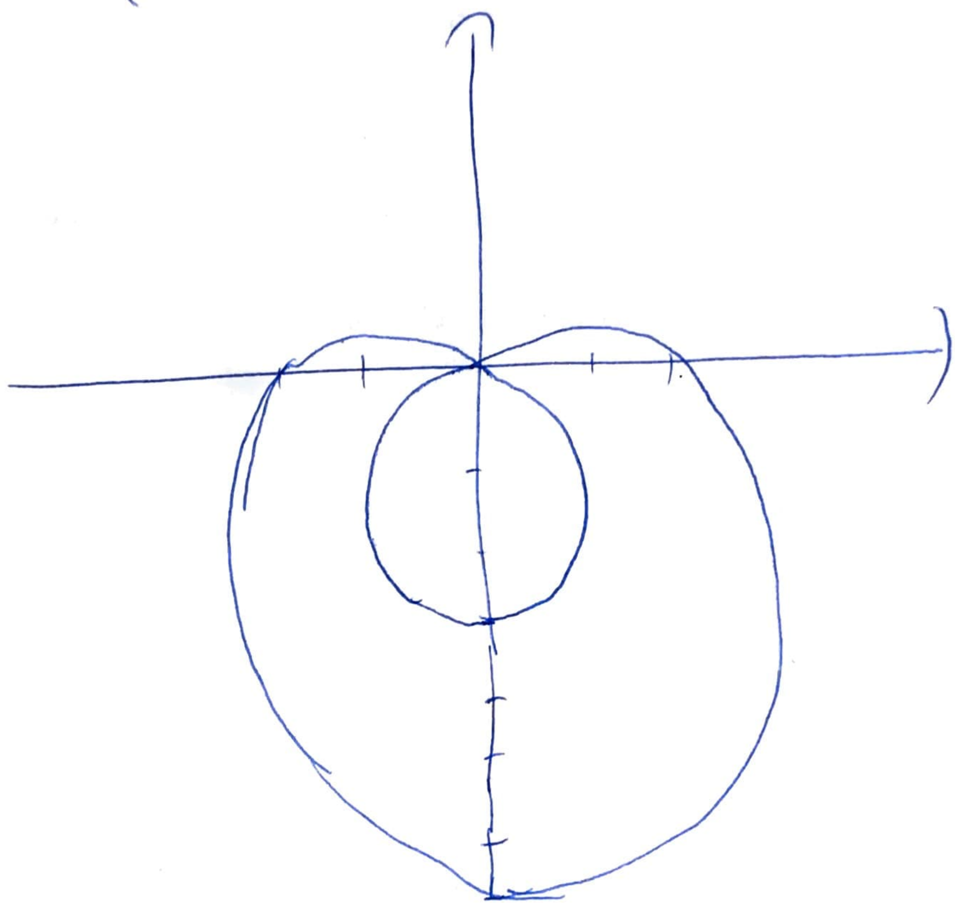


(e) $r = 1 + \cos\theta$



θ	r
0	2
$\frac{\pi}{2}$	1
π	0
$\frac{3\pi}{2}$	1

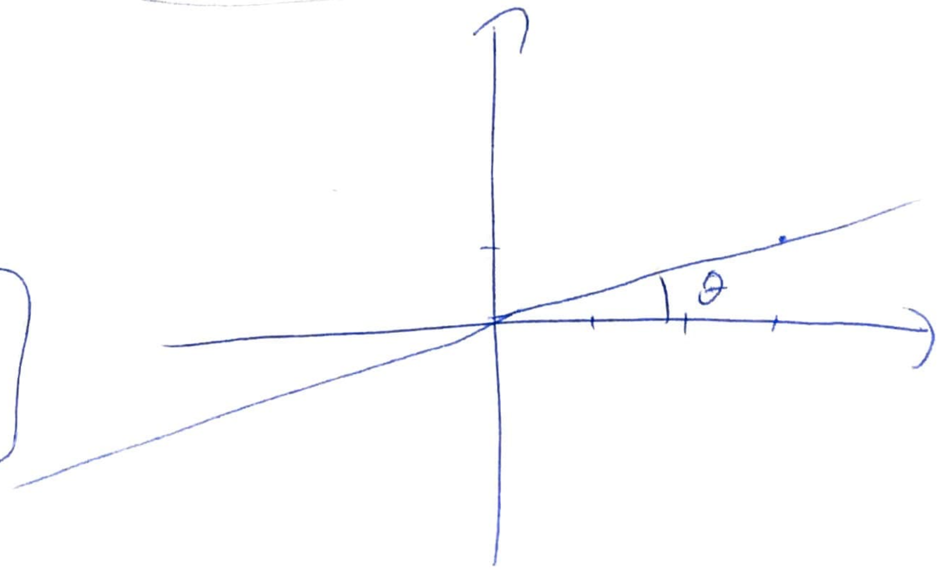
(f) $r = 2 - 5\sin(\theta)$

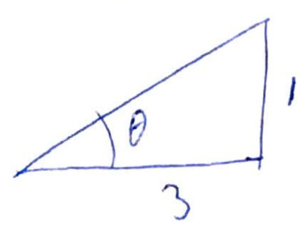


Exercise 5 :

the equation is

$$\boxed{\theta = \tan^{-1}\left(\frac{1}{3}\right)}$$





Exercise 6:

(a) $x^2+y^2=9$

we have $(r\cos\theta)^2+(r\sin\theta)^2=9 \Leftrightarrow$

$r^2(\cos^2\theta+\sin^2\theta)=9 \Leftrightarrow r^2=9 \Leftrightarrow$

$\boxed{r=3}$ (circle with radius 3)

(b) $x+y=4$

$r\cos\theta+r\sin\theta=4 \Leftrightarrow r(\cos\theta+\sin\theta)=4 \Leftrightarrow$

$$\boxed{r=\frac{4}{\cos\theta+\sin\theta}}$$

(b) $x=4 \Leftrightarrow \boxed{r\cos\theta=4} \Leftrightarrow \boxed{r=\frac{4}{\cos\theta}}$

(c) $\cancel{xy=4}$ $y=4 \Rightarrow \boxed{r\sin\theta=4}$

(d) $xy=4 \Leftrightarrow r\cos\theta\, r\sin\theta=4 \Rightarrow r^2\cos\theta\cdot\sin\theta=4$

$$r=\sqrt{\frac{4}{\sin\theta\cdot\cos\theta}}$$

Exercise 7 :

$r = 2\sin\theta \Rightarrow r^2 = 2r\sin\theta = 2y \Rightarrow$

$$x^2 + y^2 = 2y \Leftrightarrow x^2 + y^2 - 2y = 0 \Rightarrow$$

$$x^2 + y^2 - 2y + 1 - 1 = 0 \Leftrightarrow$$

$$x^2 + (y-1)^2 = 1 \Rightarrow \text{center} : (0,1)$$

radius : $r = 1$.

Exercise 8 :

We will first do it in general, the distance between $(r_1, \theta_1), (r_2, \theta_2)$

(r_1, θ_1) gives $x_1 = r_1 \cos\theta_1$, $y_1 = r_1 \sin\theta_1$, $(\overset{x_1}{r_1\cos\theta_1}, \overset{y_1}{r_1 \sin\theta_1})$.

(r_2, θ_2) gives $x_2 = r_2 \cos\theta_2$, $y_2 = r_2 \sin\theta_2$, $(\overset{x_2}{r_2\cos\theta_2}, \overset{y_2}{r_2 \sin\theta_2})$

$$d = \sqrt{(x_1 - x_2)^2 + (y_1 - y_2)^2} = \sqrt{(r_1\cos\theta_1 - r_2\cos\theta_1)^2 + (r_1 \sin\theta_1 - r_2\sin\theta_2)^2}$$

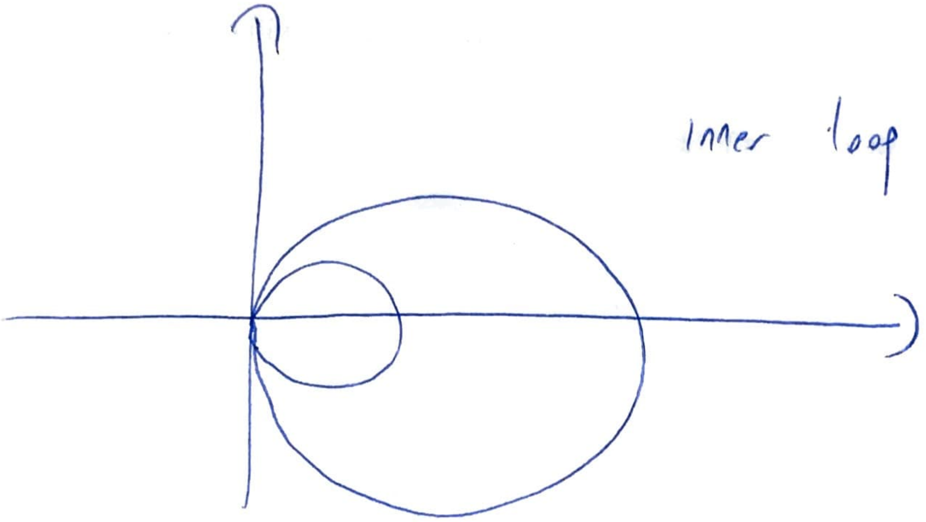
$$= \sqrt{r_1^2\cos^2\theta_1 - 2r_1 r_2 \cos\theta_1\cos\theta_2 + r_2^2\cos^2\theta_2 + r_1\sin^2\theta_1 - 2r_1r_2\sin\theta_1\sin\theta_2 + r_2^2\sin^2\theta_2} =$$

rove that

$$= \sqrt{r_1^2 + r_2^2 - 2r_1 r_2 ((\cos\theta_1\cos\theta_2 + \sin\theta_1\sin\theta_2)}$$

limacon (the name of the graph)

$r = a \pm b \sin\theta$ or $r = a \pm b \cos\theta$

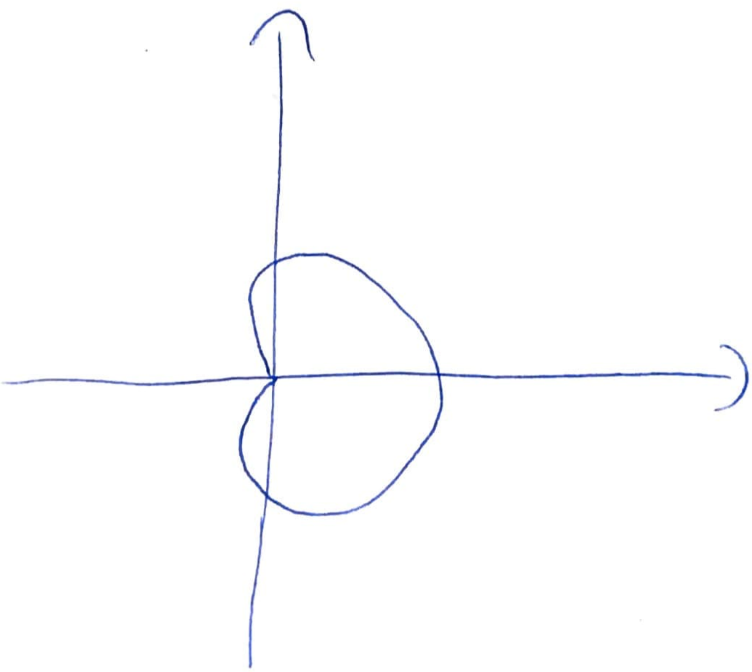


inner loop if $\frac{a}{b} < 1$

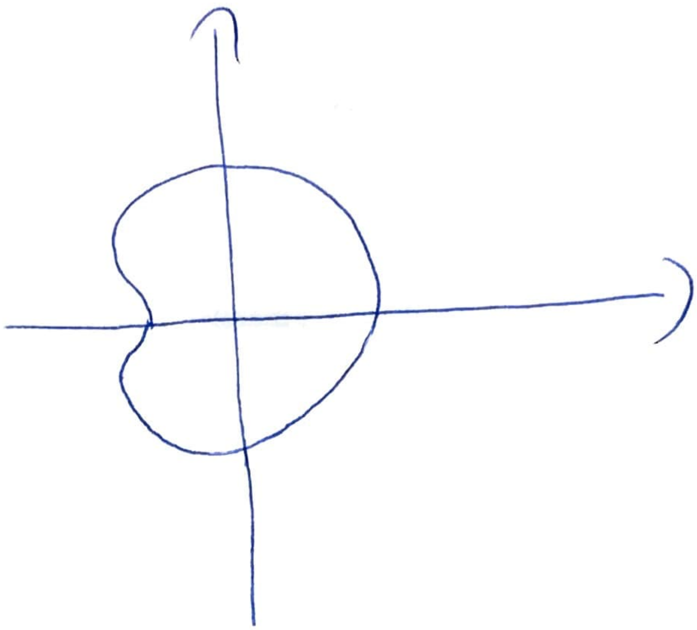
a positive

b positive

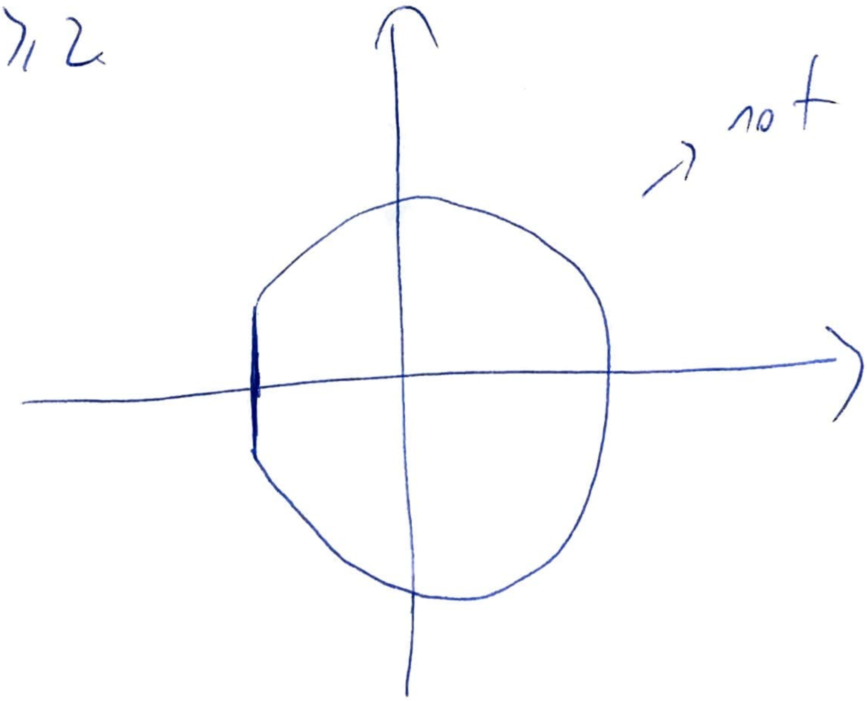
if $\frac{a}{b} = 1 \Rightarrow$ cardiod.



if $1 < \frac{d}{b} < 2$



if $\frac{d}{b} \geqslant 2$

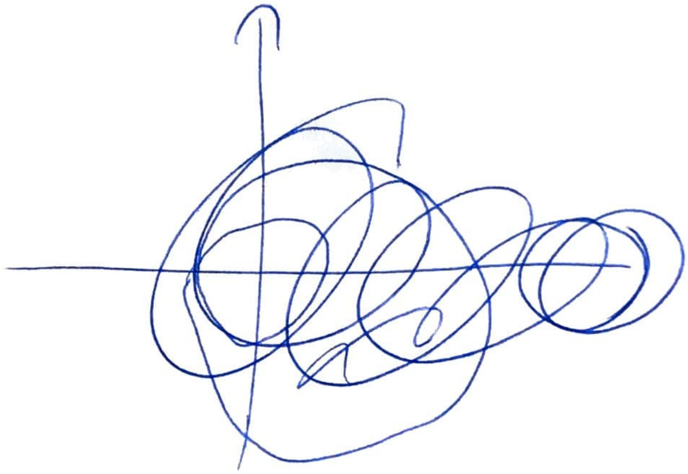


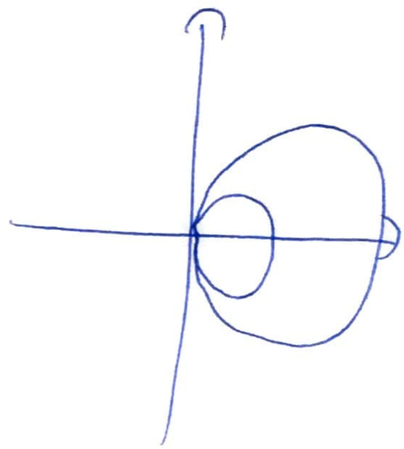
not a circle

$r = 3 + 5\cos\theta$ $\rightarrow$ type of Limacons

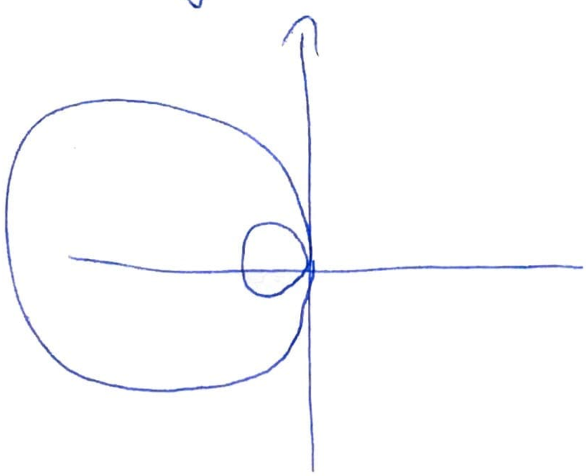
$a = 3$ $b = 5$ $\frac{a}{b} = \frac{3}{5} < 1.$

if it is $+\cos\theta$

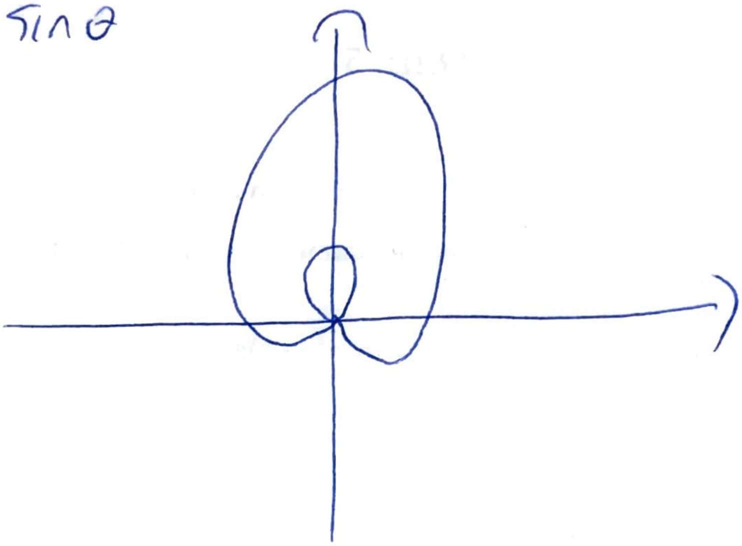




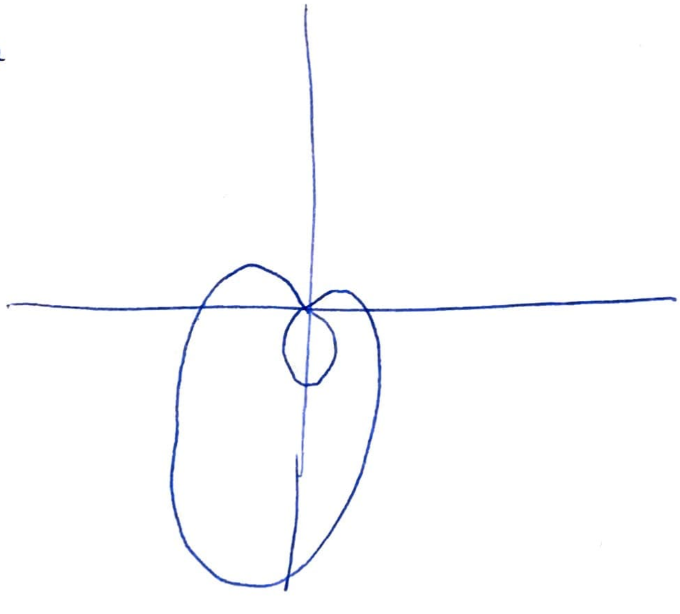
if $-\cos\theta$



if $+ \sin\theta$

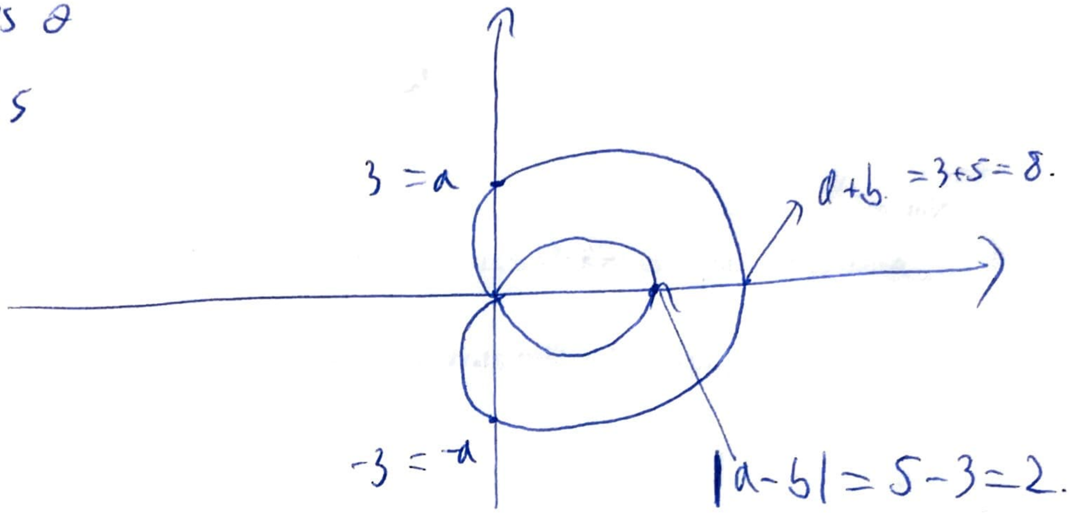


if $- \sin\theta$



$r = 3 + 5\cos\theta$

$a = 3 \quad b = 5$



$r = 2 - 5\sin\theta$

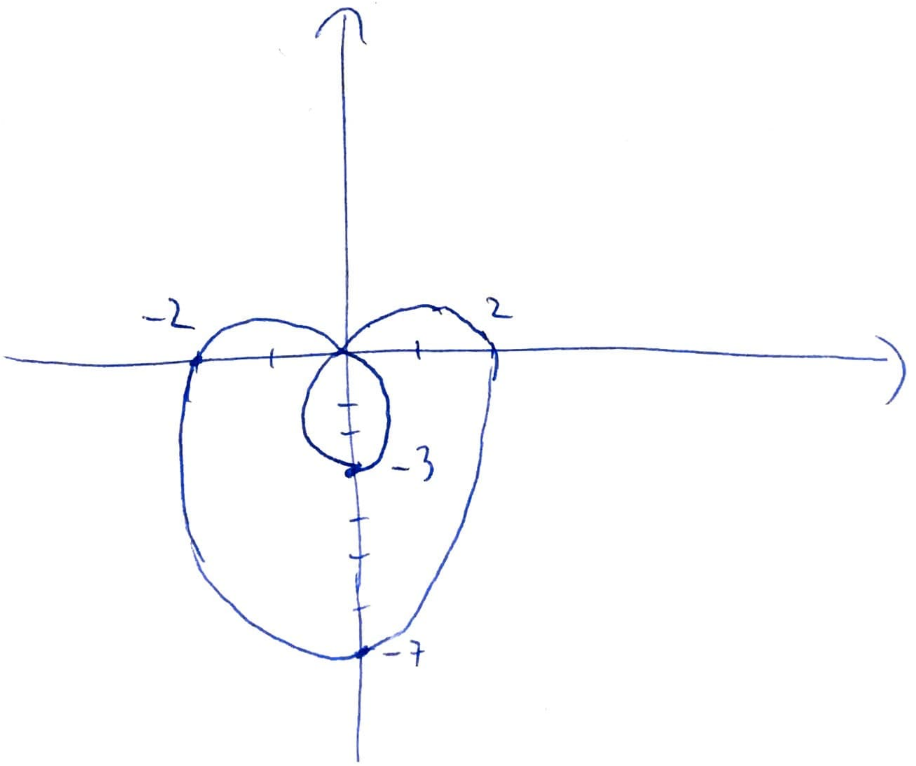
$a = 2 \ , \ b = 5$

$\frac{a}{b} = \frac{2}{5} < 1 \rightarrow$ Limacon.

we have $-\sin\theta \Rightarrow \downarrow$

$a + b = 7$

$b - a = 3.$



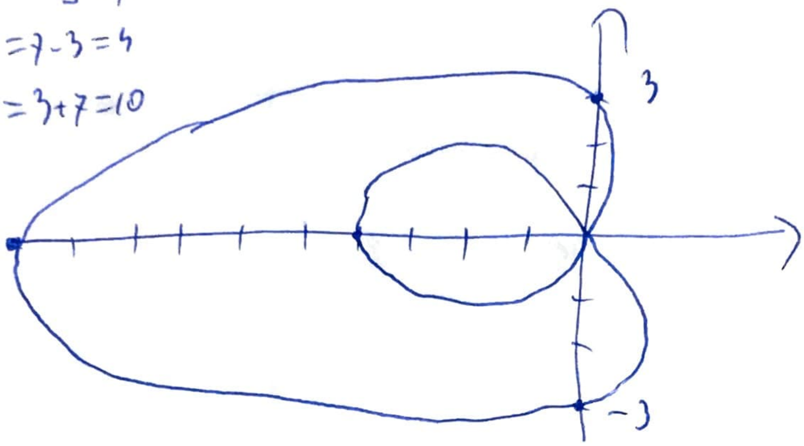
$r = 3 - 7\cos\theta$ $\qquad \frac{a}{b} = \frac{3}{7} < 1$

$a = 3$

$b = 7$

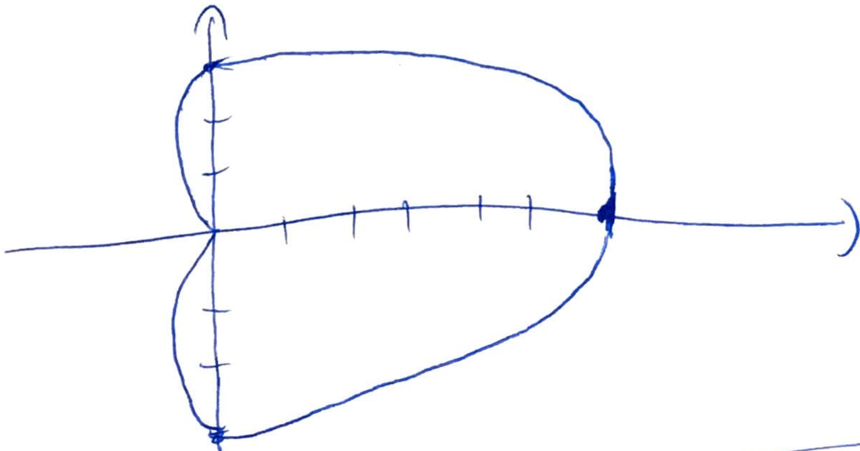
$b - a = 7 - 3 = 4$

$a + b = 3 + 7 = 10$



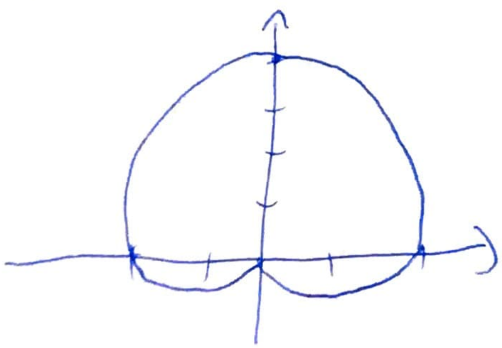
$r = 3 + 3\cos\theta$ $\Rightarrow$ $\frac{a}{b} = \frac{3}{3} = 1.$

$a + b = 6$ $\qquad \rightarrow$ Cardiod



$r = 2 + 2\sin\theta$ $\qquad \frac{a}{b} = \frac{2}{2} = 1$ $\qquad a = 2$

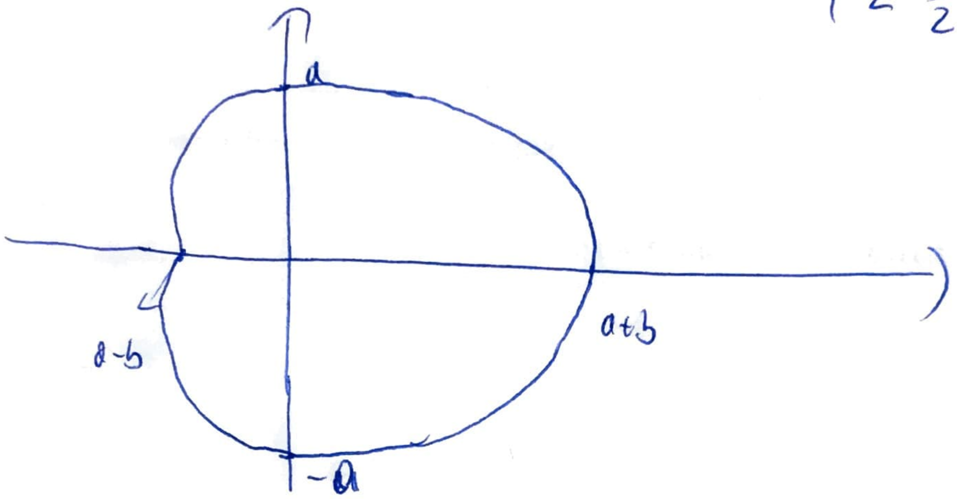
$a + b = 4$



$r = 3 + 2\cos\theta$

$\frac{a}{b} = \frac{3}{2} = 1.5$

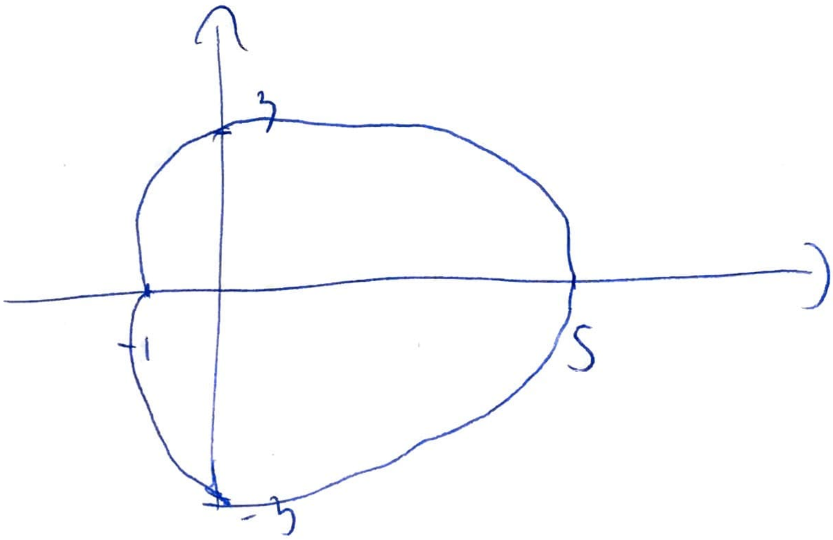
$1 < \frac{3}{2} < 2.$



$a+b = 5$

Our case

$a-b = 3-2 = 1$

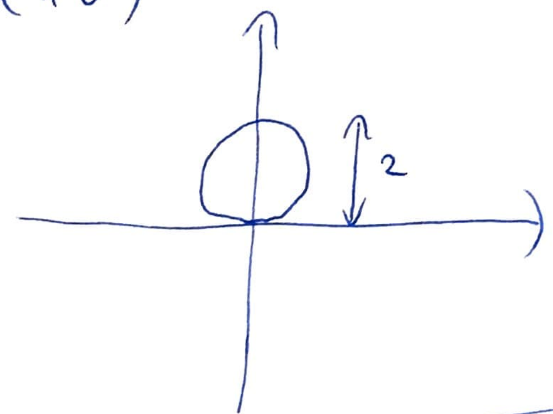


$r = a \sin n\theta$ or $r = a \cos n\theta$

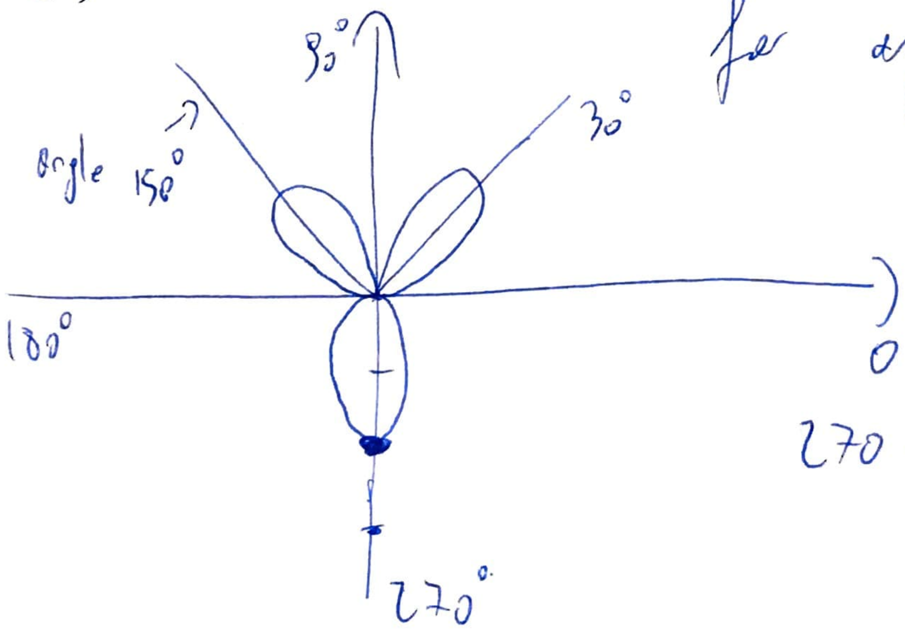
if n even # petals $= 2n$

if n odd # petals $= n$.

$r = 2\sin(1\theta)$ # $= n = 1$



$r = 2\sin(3\theta)$ $n = 3 \Rightarrow$ # petals $= 3$

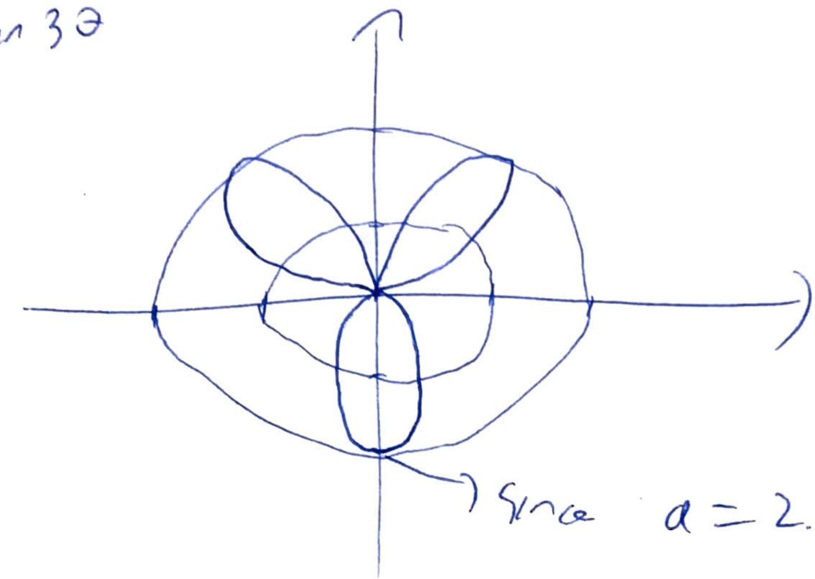


for angles

$\frac{360}{3} = 120°$.

$270 - 120 = 150$

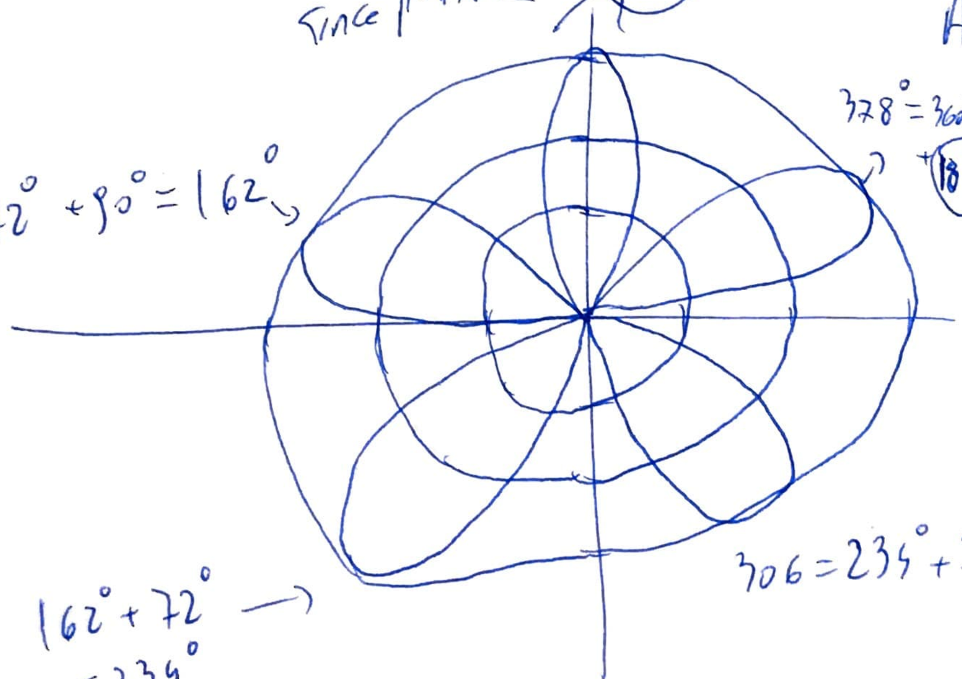
$r = 2 \sin 3\theta$



since $a = 2$.

$r = 3 \sin(5\theta)$ $\quad n = 5 \Rightarrow$ # petals = 5.

since positive 90°



Angles $\Rightarrow$ $\frac{360}{5} = 72^\circ$ apart.

$378^\circ = 360^\circ + 18^\circ$

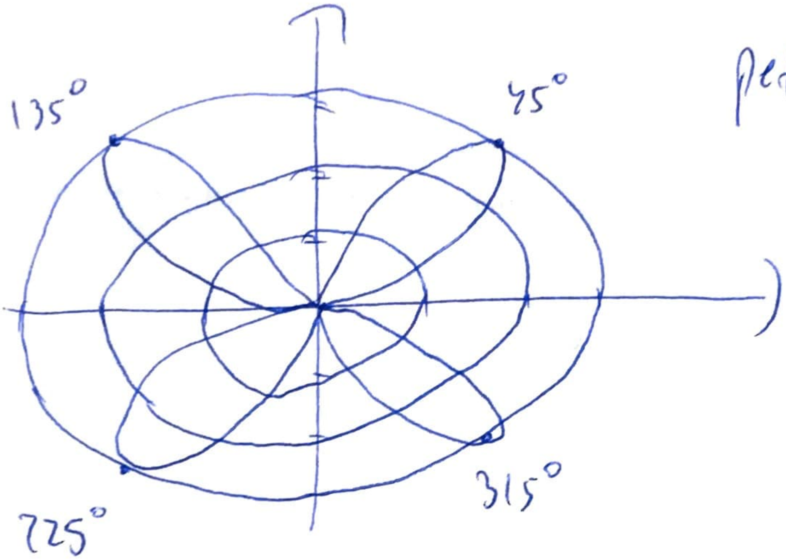
$72^\circ + 90^\circ = 162^\circ$

$306 = 234^\circ + 72^\circ$

$162^\circ + 72^\circ = 234^\circ$

$r = 3\sin(2\theta)$ $\quad n=2 \Rightarrow$ # petals $= 2n = \boxed{4}$

petals not in Ox or Oy since n is even

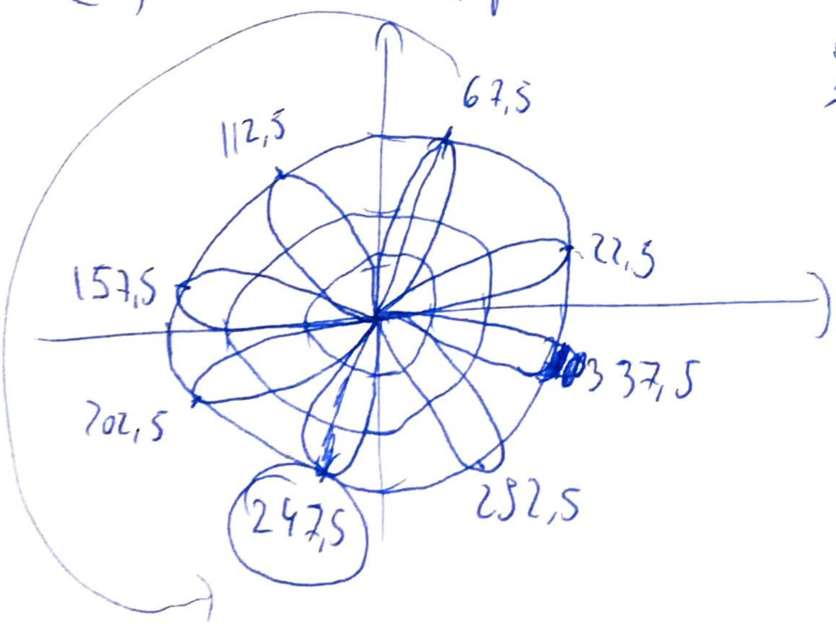


$r = -3\sin(2\theta)$ graph ~~or~~ will not change

$r = 3\sin(4\theta)$ $\quad$ #p $= 2n = 2\cdot 4 = 8.$

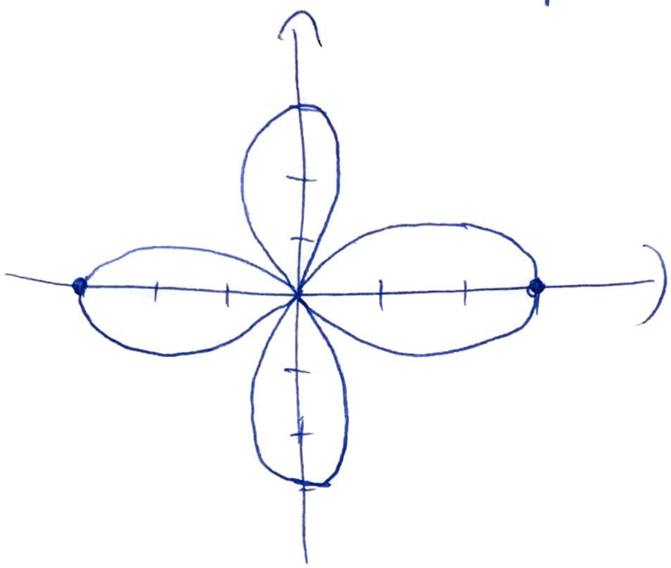
$\frac{90}{4} = 22,5$

$\frac{360}{8} = 45°$



$r = 3\cos(2\theta)$ $\qquad$ $\#p = 2n = 4.$

→ can have petals on ox or oy



Also

$r = -3\cos(2\theta)$

←

$r = 3\cos(4\theta)$ $\qquad$ $\#p = 2n = 8.$

$\frac{360}{8} = 45$