Worksheet 19;

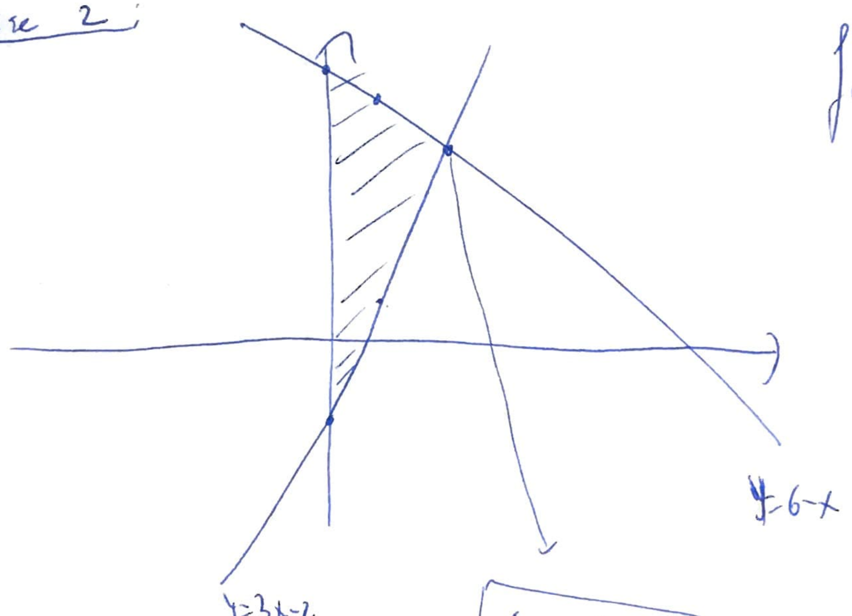
Exercise 1;

a) $V = 2\pi \int_a^b x f(x)\,dx$

b) Integrating with respect to y because you are rotating around the y-axis.

Exercise 2;

(a)



$f(x) = 6 - x$

$g(x) = 3x - 2$

$6 - x = 3x - 2$

$6 = 4x - 2$

$8 = 4x$

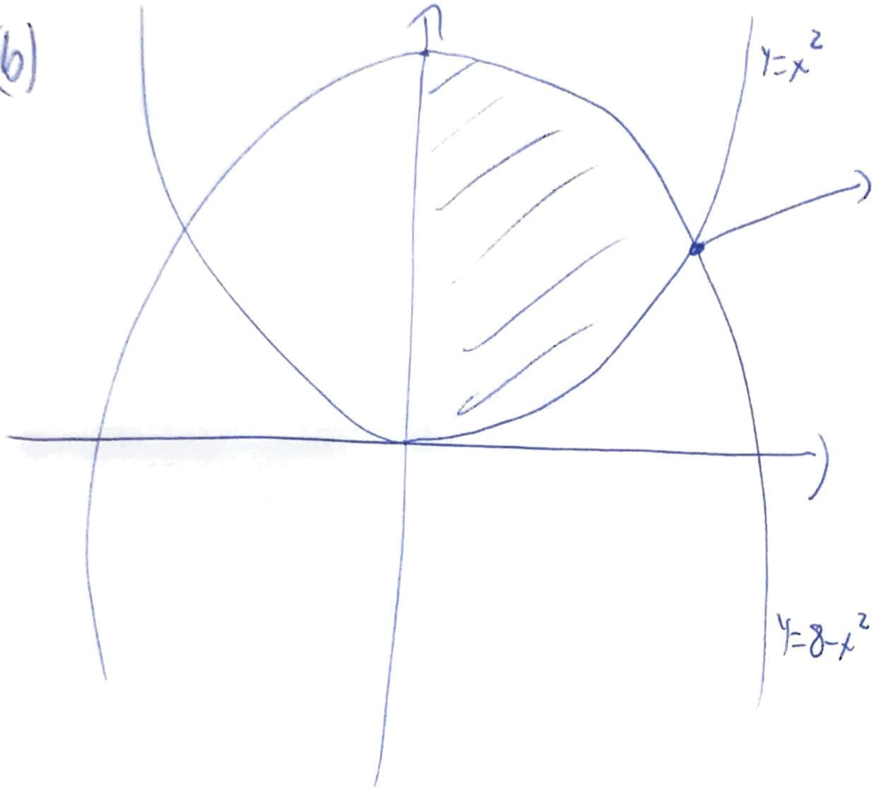
$x = 2$

$V = 2\pi \int_0^2 x(6-x-(3x-2))\,dx =$

$V = 2\pi \int_0^2 x(8-4x)\,dx =$

$V = 2\pi \int_0^2 (8x-4x^2)\,dx$

(b)



$8-x^2=x^2$

$8=2x^2$

$x^2=4$

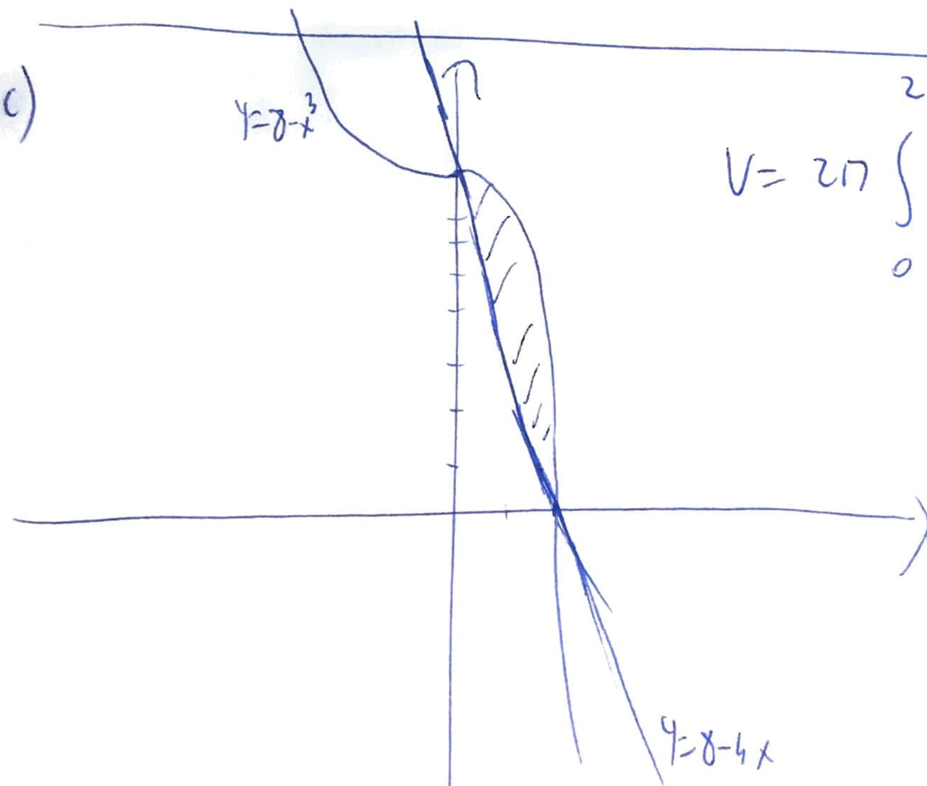
$\boxed{x=2}$

$$V=2\pi\int_0^2 x\left(8-x^2-x^2\right)dx$$

$$V=2\pi\int_0^2 x\left(8-2x^2\right)dx=$$

$$V=2\pi\int_0^2 \left(8x-2x^3\right)dx.$$

c)



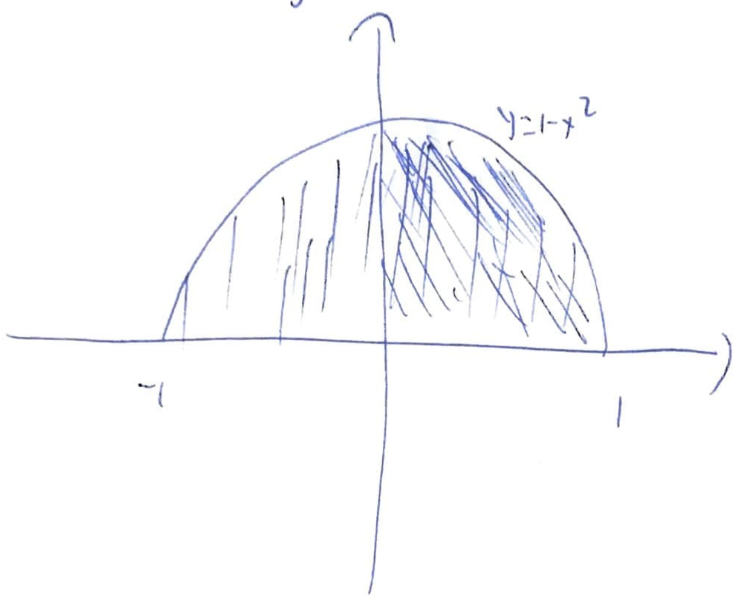
$$V=2\pi\int_0^2 x\left(8-x^3-(8-4x)\right)dx=$$

$$V=2\pi\int_0^2 x\left(4x-x^3\right)dx=$$

$$V=2\pi\int_0^2 \left(4x^2-x^4\right)dx$$

Exercise 3: Using washers or disks

(a) R is region bounded by $y = 1 - x^2$ and $y = 0$ about the x-axis



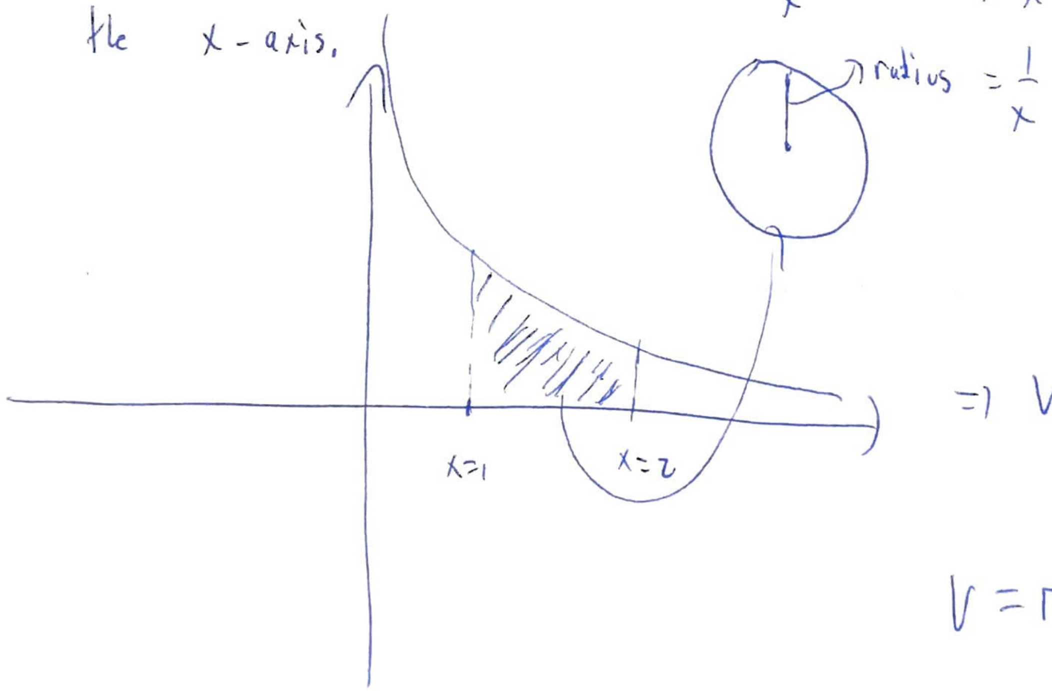
$$V = 2\pi \int_0^1 (1 - 2x^2 + x^4)\, dx$$

$$= 2\pi \int_0^1 (1 - 2x^2 + x^4)\, dx =$$

$$= 2\pi \left(x - \frac{2}{3}x^3 + \frac{x^5}{5} \right) \Big|_0^1 =$$

$$= 2\pi \left(1 - \frac{2}{3} + \frac{1}{5} \right)$$

(b) R is region bounded by $y = \frac{1}{x}$, $x = 1$, $x = 2$ and $y = 0$ about the x-axis.



$$A_0 = \pi r^2 = \pi \cdot \left(\frac{1}{x}\right)^2 = \frac{\pi}{x^2}$$

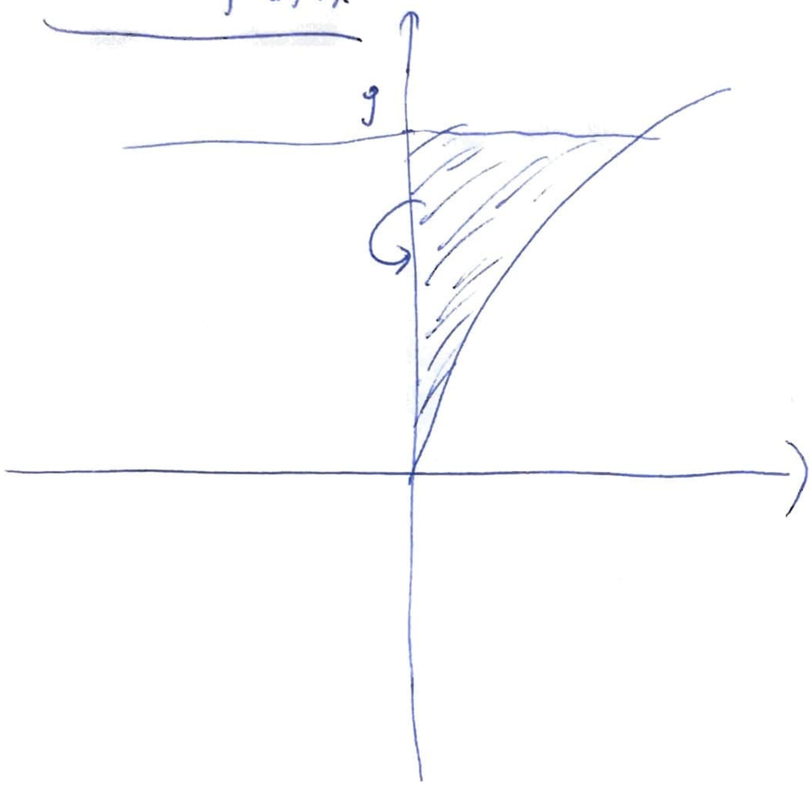
$$\Rightarrow V = \pi \int_1^2 \frac{1}{x^2}\, dx =$$

$$V = \pi \int_1^2 x^{-2}\, dx = \pi \cdot \frac{x^{-1}}{-1} \Big|_1^2$$

$$= \frac{-\pi}{x} \Big|_1^2 = \frac{-\pi}{2} + \frac{\pi}{1}$$

$$= \boxed{\frac{\pi}{2}}$$

(c) R is the region bounded by $x=2\sqrt{y}$, $x=0$ and $y=9$ about the y-axis.



$$A_0 = \pi r^2 = \pi\cdot(2\sqrt{y})^2$$

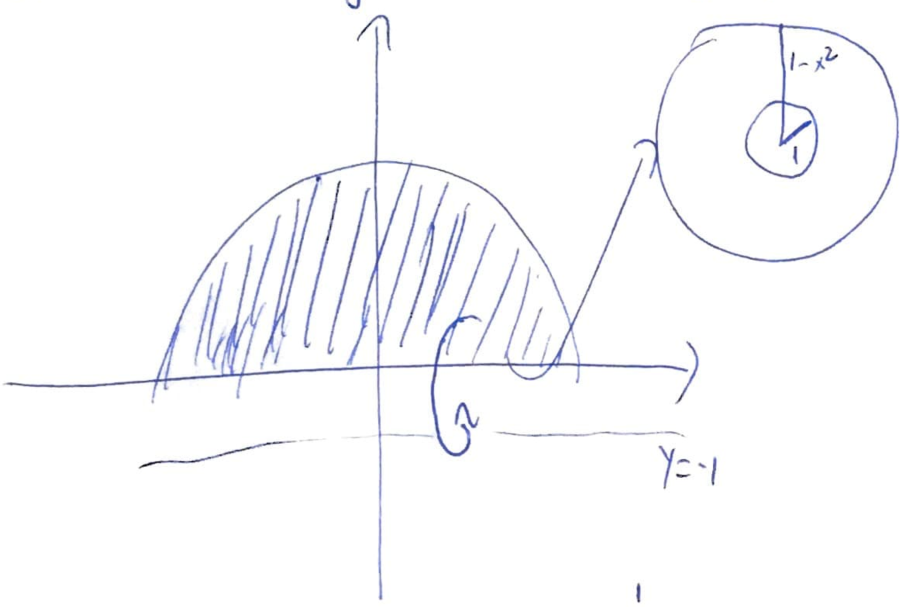
$$= 4\pi y \Rightarrow$$

$$V = \pi \int_0^9 4y\,dy$$

$$= \pi\cdot 4\frac{y^2}{2}\Big|_0^9 = \pi\cdot 2y^2\Big|_0^9$$

$$= \boxed{\pi\cdot 2\cdot 9^2}$$

(d) R is the region bounded by $y=1-x^2$ and $y=0$ about the line $y=-1$.



$$A = \pi(-1-1+x^2)^2 - \pi(-1-0)^2$$

$$= \pi(-2+x^2) - \pi =$$

$$= \pi(4-4x^2+x^4-1) =$$

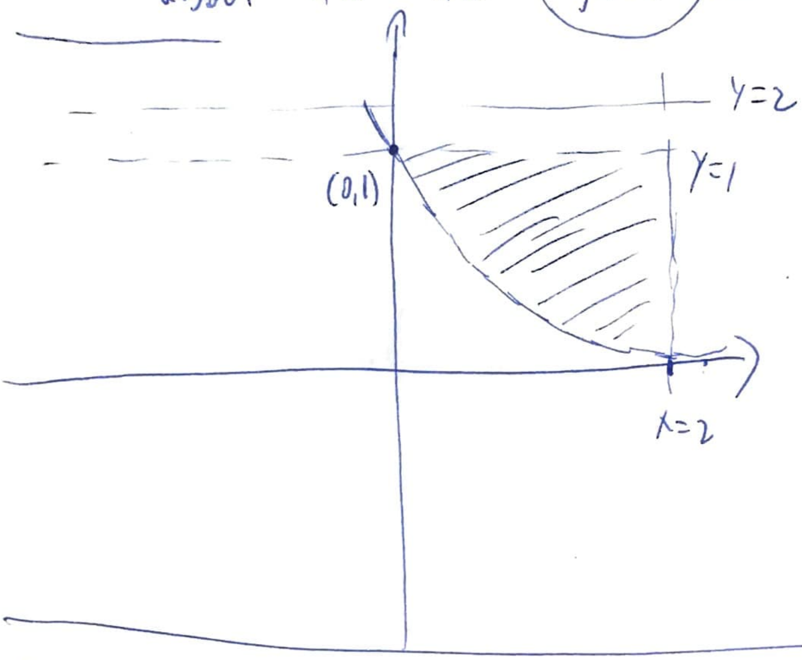
$$= \pi(3-4x^2+x^4)$$

$$V = 2\pi\int_0^1 (3-4x^2+x^4)\,dx = 2\pi\left(3x - \frac{4}{3}x^3 + \frac{x^5}{5}\right)\Big|_0^1 =$$

$$= 2\pi\left(3 - \frac{4}{3} + \frac{1}{5}\right).$$

(e) After calculating the region in part (d) is larger than part (i).

(f). R is the region bounded by $y = e^{-x}$, $y=1$ and $x=2$ about the line $y=2$.



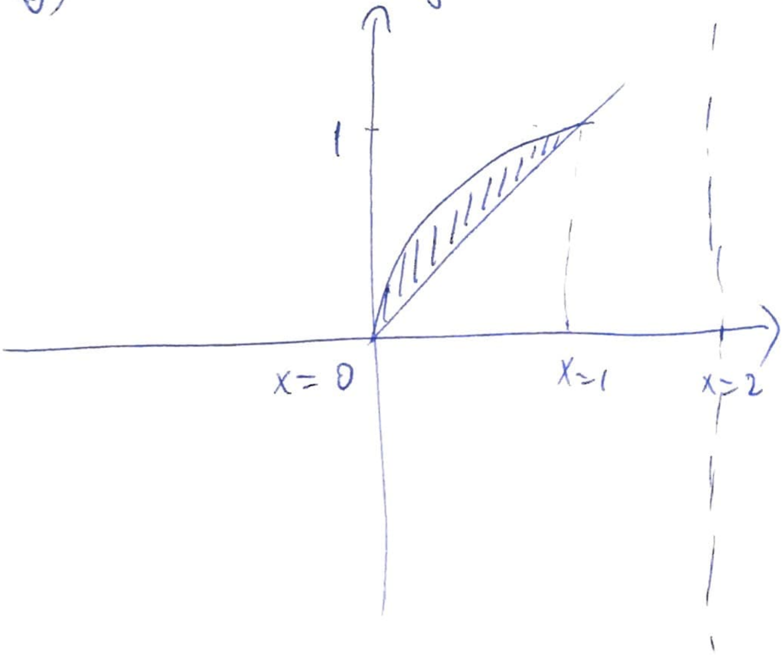
$2-1$

$$A = \pi\left((2-e^{-x})^2 - (1)^2\right)$$

$$= \pi\left((2-e^{-x})^2 - 1\right)$$

$$\Rightarrow V = \pi\int_0^1 \left((2-e^{-x})^2 - 1\right) dx.$$

(g) R is the region bounded by $y=x$, $y=\sqrt{x}$ about the line $y=2$.



$x = y^2$

$x = y$

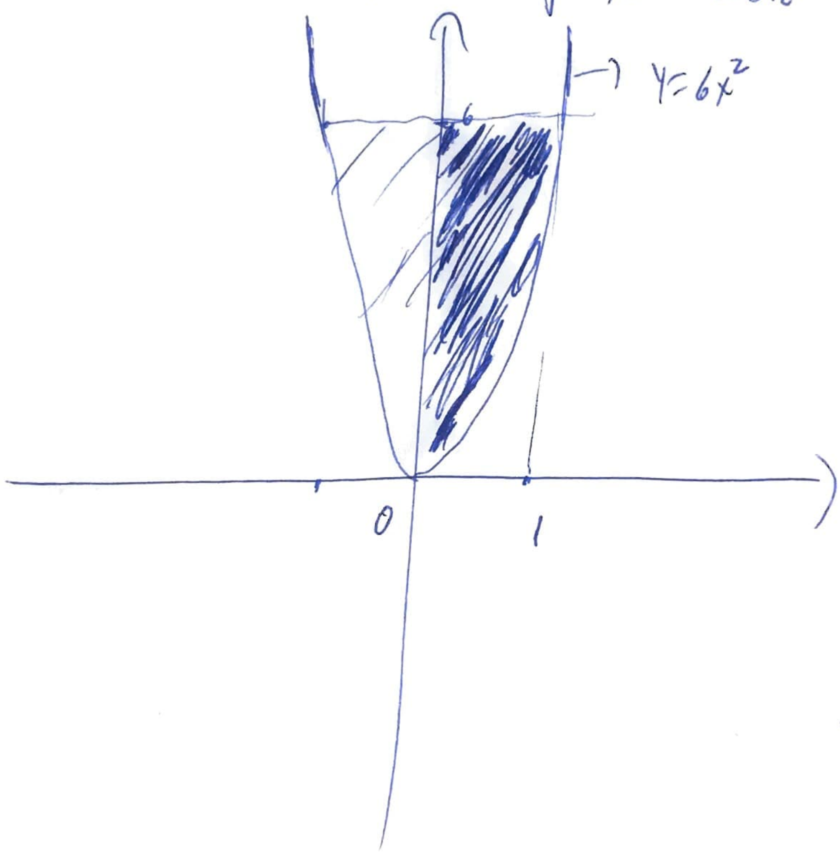
$$\Rightarrow A = \pi\left((2-y^2)^2 - (2-y)^2\right)$$

$$\Rightarrow V = \pi\int_0^1 \left((2-y^2)^2 - (2-y)^2\right) dy$$

Exercise 4;

Generated by $y=6x^2$, $0 \le x \le 1$ about the y-axis.

First we need to find the volume and then use the fact that Todo is extracted from the glass through a straw at the rate of $1/2$ cubic inches per second.



$$V = 2\pi \int_0^1 x(6-6x^2)\,dx$$

$$= 2\pi \int_0^1 (6x - 6x^3)\,dx$$

$$= 2\pi \cdot \left(6\frac{x^2}{2} - 6\frac{x^4}{4}\right)\Bigg|_0^1$$

$$= 2\pi\left(6\cdot\frac{1}{2} - 6\cdot\frac{1}{4}\right) =$$

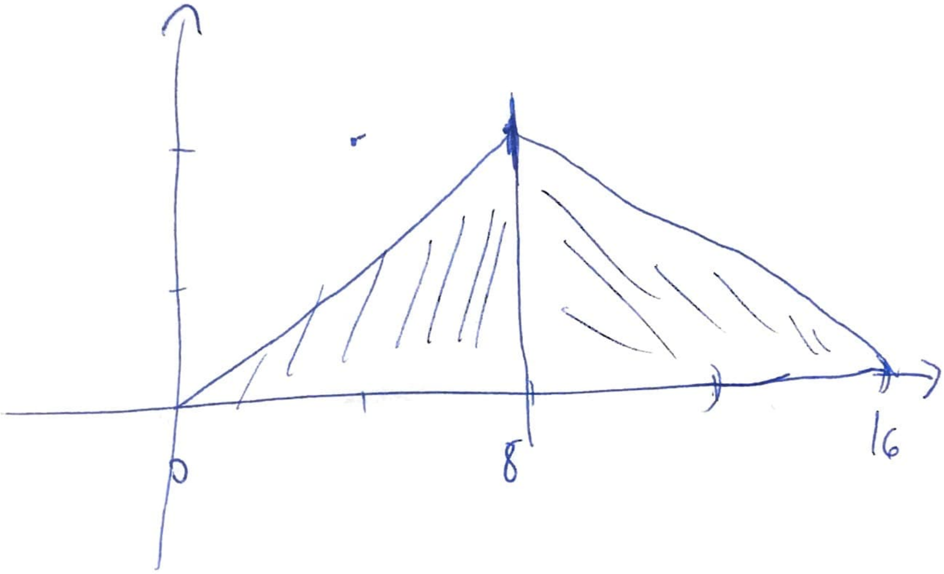
$$= 2\pi\left(3 - \frac{3}{2}\right) = 2\pi\cdot\left(\frac{6-3}{2}\right)$$

$$= 3\pi \text{ inches}^3$$

then we just multiply by $\frac{1}{2}$.

Extra exercise:

Find the volume of the solid obtained by rotating the region enclosed by the graphs of $f(x) = 8 - |x-8|$, $y=0$ about the y-axis (using Shells).



for $x \in [0,8] \Rightarrow |x-8| = -(x-8) = -x+8.$

$\Rightarrow f(x) = 8 - (-x+8) = 8 + x - 8 = x.$

for $x \in [8,16] \Rightarrow |x-8| = x-8$

$\Rightarrow f(x) = 8 - (x-8) = 16 - x. \Rightarrow$

$$V = 2\pi \int_0^8 x \cdot x \, dx + 2\pi \int_8^{16} x \cdot (16-x) \, dx$$