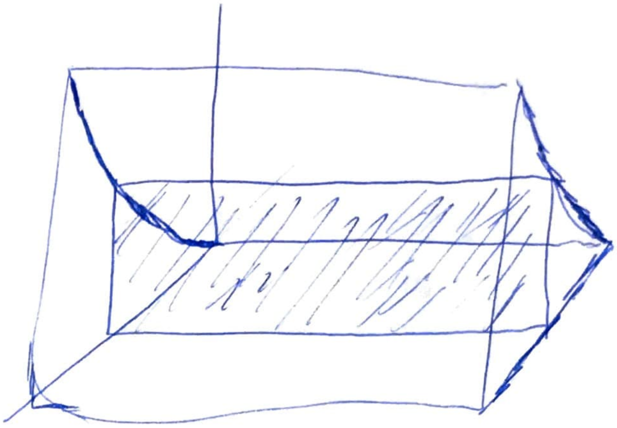
Worksheet 18 :

Exercise 2 :

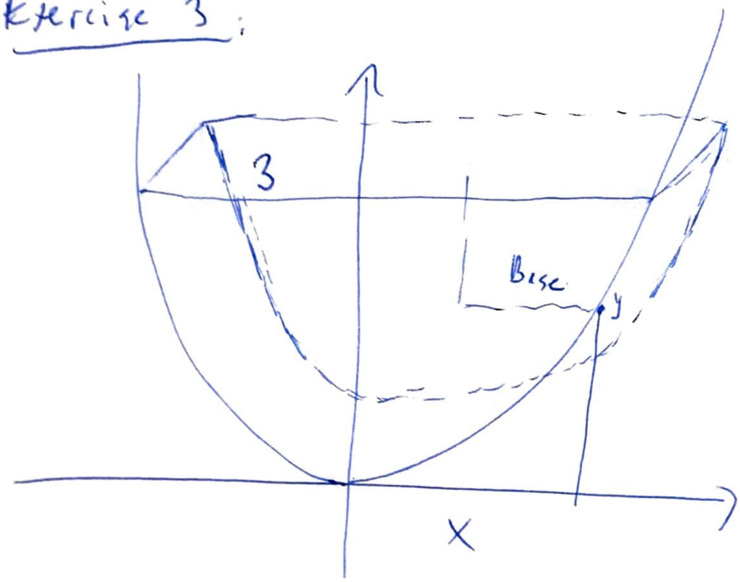
$A(x) = x^2$, $V = \int_0^l x^2\,dx = \frac{1}{3}x^3\Big|_0^l = \frac{l}{3}$



Exercise 1 :

Volume $= \int A(x)\,dx$

Exercise 3 :

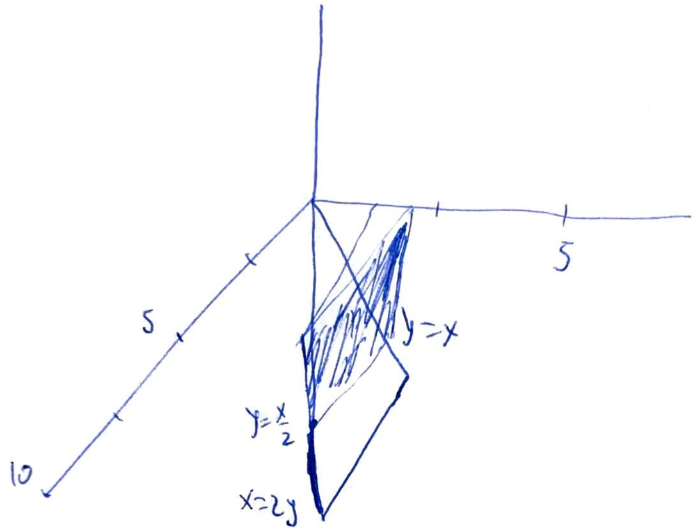


Integrating with respect to y, so the area of the circles need to be in terms of y.

So $x = 2\sqrt{y}$

$$\int_0^3 4y\,dy = 2y^2\Big|_0^3 = 18.$$

Exercise 4:

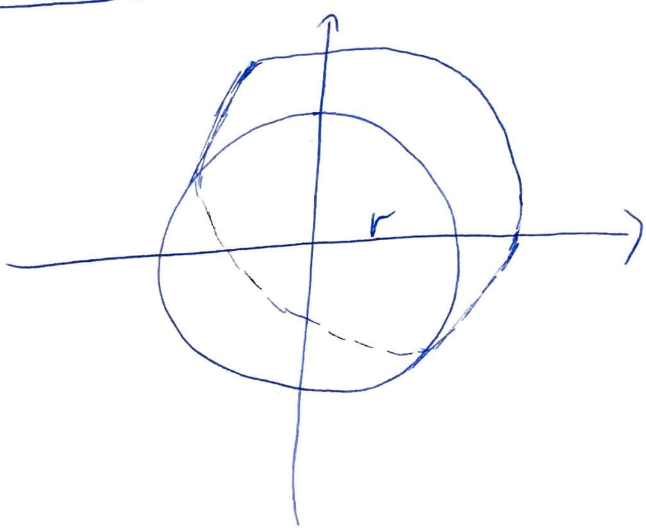


$A(y) = (2y - y)^2 = y^2$

$$v = \int_0^5 y^2 dy = \frac{1}{3} y^3 \Big|_0^5$$

$$= \frac{125}{3}$$

Exercise 5:



$x^2 + y^2 = r^2$ area in terms of x

$y^2 = r^2 - x^2 \Rightarrow y = \sqrt{r^2 - x^2}$

Side length $= 2y = 2\sqrt{r^2 - x^2}$

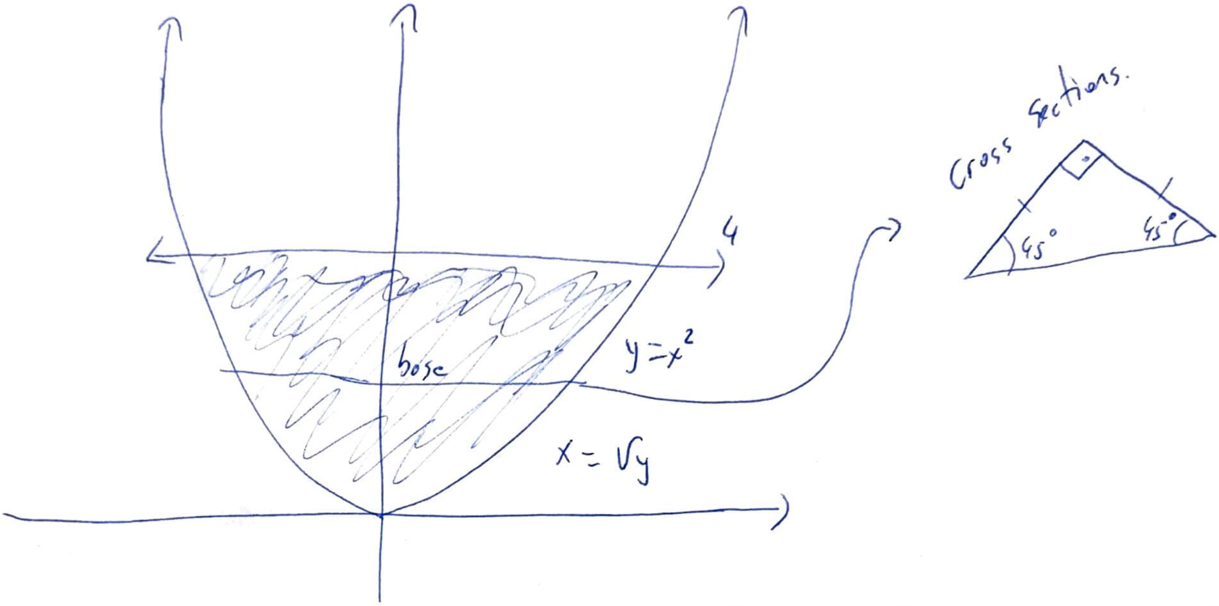
$$\int_{-r}^{r} A(x)dx = 2\int_0^r (4r^2 - 4x^2)dx =$$

$$= 2\left(4r^2 x - \frac{4}{3}x^3\right)\Big|_0^r =$$

$$= 2\left(4r^3 - \frac{4}{3}r^3\right) =$$

$$= 2\left(\frac{8}{3}r^3\right) = \frac{16}{3}r^3.$$

Exercise 6 :



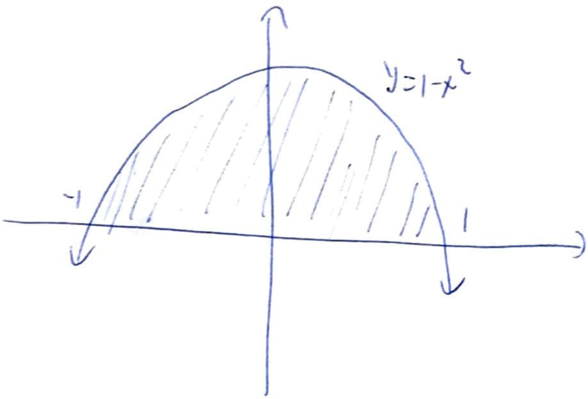
hypotenuse $= 2\sqrt{y}$, $s = \frac{h}{\sqrt{2}} = \frac{2\sqrt{y}}{\sqrt{2}} = \sqrt{2}\sqrt{y} = \sqrt{2y}$

$A(y) = \frac{1}{2}s^2 = \frac{1}{2}(2y) = y.$

$V = \int_0^4 A(y)\,dy = \int_0^4 y\,dy = \frac{1}{2}y^2\Big|_0^4 = \frac{1}{2}\cdot 16 = 8.$

Exercise 8:

(a) R is the region bounded by $y=1-x^2$ and $y=0$; about the x-axis



$$2\pi \int_0^1 \left((1-2x^2)+x^4\right) dx =$$

$$= 2\pi \left(x - \frac{2}{3}x^3 + \frac{1}{5}x^5\right)\Big|_0^1 =$$

$$= 2\pi \left(1 - \frac{2}{3} + \frac{1}{5}\right) \approx 3,351.$$

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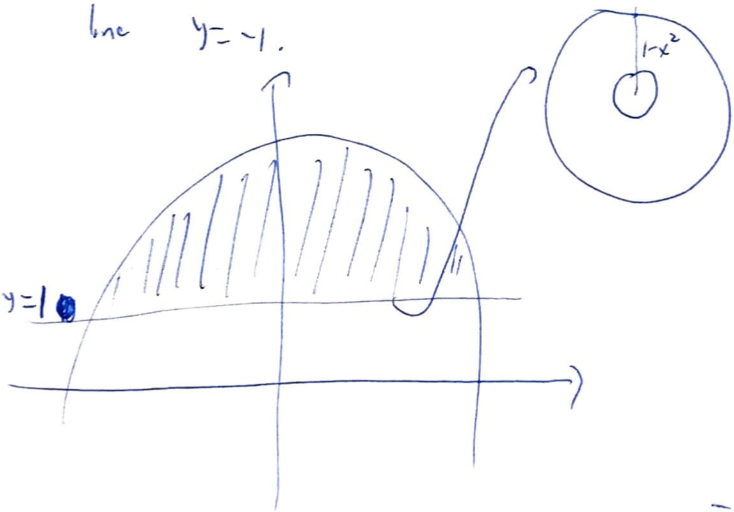
b) R is the region bounded by $y=1-x^2$ and $y=0$; about the line $y=-1$.



$$A = \pi(-1-1+x^2)^2$$

$$-\pi(-1-0)^2 =$$

$$= \pi(-2+x^2)^2 - \pi =$$

$$= \pi(4-4x^2+x^4-1)$$

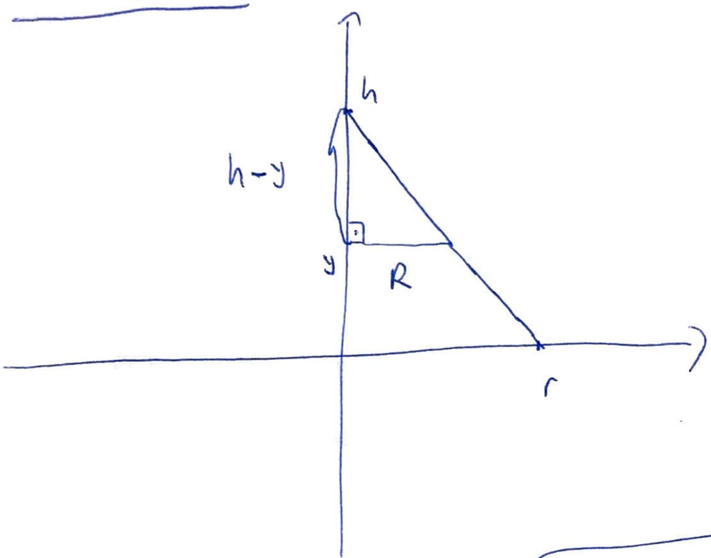
$$= \pi(3-4x^2+x^4)$$

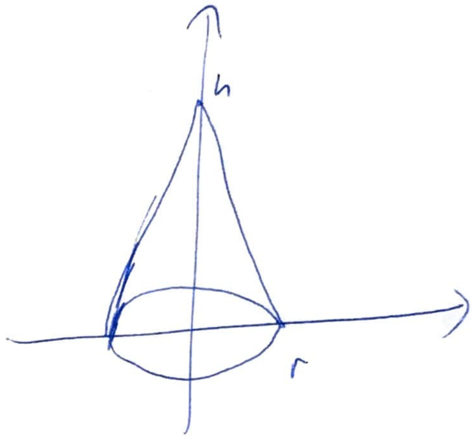
$$2\pi \int_0^1 (3-4x^2+x^4)\,dx =$$

$$= 2\pi \cdot \left(3x - \frac{4}{3}x^3 + \frac{1}{5}x^3\right)\Big|_0^1 = 2\pi\left(3 - \frac{4}{3} + \frac{1}{4}\right) \approx 11,7286.$$

(c) The region in part b) has a bigger volume.

Exercise 3:





$$\frac{R}{h-y} = \frac{r}{h} \Rightarrow \boxed{R = \frac{r}{h}\cdot(h-y)}$$

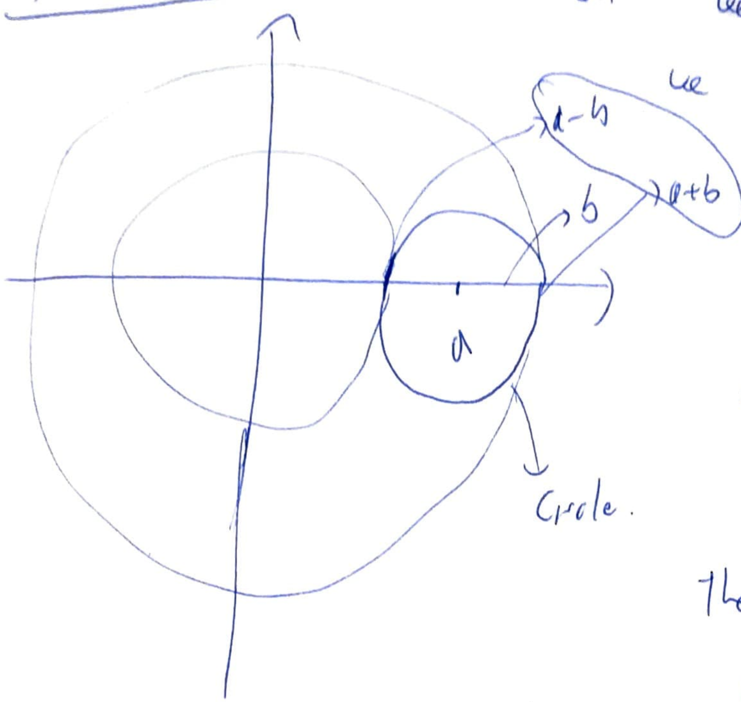
$$A(y) = \pi R^2 = \pi\cdot\left(\frac{r}{h}(h-y)\right)^2 = \pi\frac{r^2}{h^2}\left(h^2 - 2hy + y^2\right)$$

$$V = \int_0^h \pi\cdot\frac{r^2}{h^2}\left(h^2 - 2hy + y^2\right)dy = \pi\frac{r^2}{\cancel{h^2}}\cancel{h^2}\int_0^h dy - \frac{\pi r^2}{h^{\cancel{2}}}\cdot 2\cancel{h}\int_0^h y\,dy$$

$$+ \frac{\pi r^2}{h^2}\int_0^h y^2\,dy = \pi r^2\cdot(h-0) - \frac{\pi r^2}{h}\cdot\frac{y^2}{2}\Big|_0^h + \frac{\pi r^2}{h^2}\frac{y^3}{3}\Big|_0^h$$

$$= \boxed{\pi r^2\cdot h - \frac{\pi r^2}{h}\left(\frac{h^2}{2} - 0\right) + \frac{\pi r^2}{h^2}\cdot\frac{h^3}{3}}$$

Exercise 10 :



When we take the integral we need to start from $a-b$ $\to$ $a+b$ for the integral.

the circumference of the circles inside is $C = 2\pi x$.

The height of the torus is given by the formula

$h = 2y = 2\sqrt{b-(x-a)^2}$ and so we have

$$V = \int_{a-b}^{a+b} 2\pi \cdot 2x\sqrt{b^2-(x-a)^2}\,dx$$

let $x = bv + a$ and so $dx = b\,dv$ and

$a-b \longrightarrow -1$

$a+b \longrightarrow 1$

$$V = 4\pi b^2 \int_{-1}^{1} (bv+a)\sqrt{1-v^2}\,dv = 4\pi b^2\left(b \cdot 0 + a\,\frac{\pi}{2}\right) = 2\pi b^2 a$$

we used the fact that since $v\sqrt{1-v^2}$ is odd function the integral is zero.