Worksheet 12:

Exercise 1:

a) $a_n = \frac{n}{3n+1}$, the sequence $\{a_n\}$ converge since $\lim_{n\to\infty} \frac{n}{3n+1} = \frac{1}{3}$

since $\lim_{n\to\infty} a_n = \frac{1}{3} \neq 0$ then $\sum_{n=1}^{\infty} a_n$ diverge.

b) $\sum_{n=1}^{\infty} \frac{1}{n}$.

c) No, Divergence Test.

d) It converges absolutely if $\sum_{n=1}^{\infty} |a_n|$ converges.

if $\sum_{n=1}^{\infty} a_n$ converges but $\sum_{n=1}^{\infty} |a_n|$ diverges then converges conditionally.

e) $\sum_{n=1}^{\infty} a_n$, if a_n is decreasing sequence of positive terms with $\lim_{n\to\infty} a_n = 0$ then $\sum_{n=1}^{\infty} (-1)^n a_n$ converges.

f)

Exercise 2:

Since $\frac{1}{n} \xrightarrow[n\to\infty]{} 0$, the alternating series converges.

Exercise 3:

a) $\sum_{n=1}^{\infty} (-1)^n \frac{\sqrt{n}}{1+2n}$, $\lim_{n\to\infty} \frac{\sqrt{n}}{1+2n} = 0$. $f(x) = \frac{\sqrt{x}}{1+2x}$

$f'(x) = \frac{\frac{1}{2}x^{-\frac{3}{2}}(1+2x) - 2\cdot x^{\frac{1}{2}}}{(1+2x)^2} < 0$ for $x \geq 1$.

Converges.

b) $\sum_{n=2}^{\infty} (-1)^n \frac{1}{\ln(n)}$ $\lim_{n\to\infty} \frac{1}{\ln(n)} = 0$, $f(x) = \frac{1}{\ln(x)}$

$f'(x) = \frac{\frac{1}{x}}{(\ln(x))^2} = \frac{1}{x\ln(x)}$.

$\Rightarrow$ converges.

Alternating Series test.

c) $\sum_{n=1}^{\infty} \frac{\cos(n\pi)}{n^{2/3}} = \sum_{n=1}^{\infty} \frac{(-1)^n}{n^{2/3}}$ converges by AST.

d) $\sum_{n=1}^{\infty} \frac{3^n}{4^n+5^n} \leq \sum_{n=1}^{\infty} \frac{3^n}{5^n}$ $\Rightarrow$ Converges.

$\frac{3^n}{4^n+5^n} \leq \frac{3^n}{5^n} = \left(\frac{3}{5}\right)^n$.

e) $\sum_{n=1} (-1)^n \cdot \frac{n}{\ln(n)}$. , let $a_n = \frac{n}{\ln(n)}$, $f(x) = \frac{x}{\ln(x)}$.

$\lim_{x\to\infty} \frac{x}{\ln x} \overset{\text{L'hopital}}{=} \lim_{x\to\infty} \frac{1}{\frac{1}{x}} = \lim_{x\to\infty} x = \infty$. By Test for Divergence it diverges..

f). $\sum_{n=1}^{\infty} \left(\frac{-5}{18}\right)$ → diverges. $\lim_{n\to\infty} \frac{-5}{18} = -\frac{5}{18} \neq 0$.

Exercise 4 :

a) $\sum_{n=1}^{\infty} \frac{(-0,8)^n}{n!}$ => four decimal we need to insure error 0.0001, check terms.

$a_4 = \frac{0,4096}{24} = 0,017$ $a_6 = \frac{0,2621}{720} = 0,00036$

$a_8 = \frac{0.1678}{40320} = 0,000004$ $a_7 = \frac{0,2097}{5040} = 0,00004$

$$\boxed{S_6 = \frac{-1}{1} + \frac{(0,8)^2}{2} - \frac{(0,8)^3}{3!} + \frac{(0,8)^4}{4!} - \frac{(0,8)^5}{5!} + \frac{(0,8)^6}{6!}}$$

b) $\sum_{n=1}^{\infty} (-1)^{n-1} \frac{n}{8^n}$ Estimate sum correct to four decimal places.

$\frac{n}{8^n} = a_n$ ensure error $0,0001$,

$a_6 = \frac{6}{8^6} = 0,00002$ $\quad a_5 = 0,00015.$

$$\boxed{S_5 = \frac{1}{8} - \frac{2}{8^2} + \frac{3}{8^3} - \frac{4}{8^4} + \frac{5}{8^5}}$$

Exercise 5 :

$\sum_{n=1}^{\infty} (-1)^n \frac{1}{(2n)!}$ $\quad |\text{error}| < 0,00005.$

$|S - S_k| < a_{k+1} < 0,00005.$

$a_6 = \frac{1}{(12)!} = \frac{1}{479001600} = 0,000000002.$

$a_3 = \frac{1}{6!} = \frac{1}{720} = 0,0014$, $\quad a_5 = \frac{1}{10!} = \frac{1}{3628800} =$

$= 0,000000276.$

$a_4 = 0,000025 \Rightarrow \boxed{S_3 = \frac{-1}{2} + \frac{1}{4!} - \frac{1}{6!}}$